CONTINUOUS BEAMS IN REINFORCED CONCRETE

BURNARD GEEN, A.M.I.C.E., M.S.E., M.C.I.

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BY

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PREFACE

AS far as the Author is aware, there has not been published up to the present time, in a simple form, any method by which the bending moments and shearing forces in continuous reinforced concrete beams may be readily calculated; and by which, at the same time, the difference in bending moment between any two points in a beam may be readily ascertained for the purpose of determining the required number and disposition of stirrups or binding in such beam.

The diagrams and tables given in this book contain all the necessary information to permit of the rapid calculation of the maximum possible bending moments, vertical and horizontal shearing forces, and stirrups or binding, for any number of equal continuous spans, with any possible arrangement of loading of complete spans for all the types of loading generally met with in practice.

The present method of determining the sections of reinforced concrete beams is, at the best, to some extent 'rule of thumb;' and while regulations of various governing or influential bodies applying to the calculation of reinforced concrete structures do exist, they are guardedly expressed, and are of little use in dealing with the majority of ordinary cases of loading met with in everyday practice.

The recent Report of the Institution of British Architects, for example, recommends the assumption of bending moments amounting to $\frac{w\,L^2}{12}$ at the centre of the span and at the adjacent supports, where the beams are equally loaded with uniformly distributed loads, and are continuous over three or more equal spans.

The Report goes on to recommend that where the spans are of unequal length, or where the loads are not uniformly distributed, that more exact calculations should be made; but the Report does not make any suggestion as to what form these should take, beyond what may be gathered from the following extract :—

'If the bending moments are calculated by the ordinary theory of continuous beams, it should be remembered that the supports are usually assumed level, and

if this is not the case, or the supports sink out of level, the bending moments are altered.'

The statement that the bending moments are affected if the supports are not built level is not strictly correct, and the ordinary theory does apply, provided that the supports are fitted to the profile of the beam in its normal unstrained condition.

The important questions of the bending moments and shearing forces in continuous beams, under point loads, and the increased reactions which occur, have not previously received the attention they deserve, in view of the fact that they occur in practically every construction which is carried out in reinforced concrete.

It is hoped, therefore, that the information contained in this book may be of considerable use to engineers, architects, and others engaged in constructional work, as well as to students, especially in view of the fact that the calculations necessary to arrive at the limiting maximum moments and shears for any series of equal spans with any given type of loading are very lengthy and tedious, and that the necessary time for making lengthy calculations can generally ill be spared in actual practice.

For the particularly simple statement of the 'Theorem of Three Moments,' which is contained in Chapter II. and Appendix I., and is believed to be to some extent original, the Author is indebted to Mr. E. C. R. Nelson, to whom he wishes to express his thanks; as also to Mr. B. V. Richardson, for much patient labour in assisting him in the preparation of the diagrams and tables.

B. G.

King's Court, Broadway,
Westminster, S.W.,
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CONTINUOUS BEAMS IN REINFORCED CONCRETE

CHAPTER I.

THE calculations involved in determining the sections of reinforced concrete beams are necessarily divided into two parts, viz.:

(*a*) Calculation of bending moments and shearing forces.
(*b*) Calculation of resisting moments and shearing resistances.

Much has been written about the latter and but little about the former, though the former is, if anything, the more important.

If the assumptions as to the moments and shears are inaccurate to 10% or even 20%, it is ridiculous to attempt to give great accuracy in the resisting moments and shearing resistances.

The purpose of this book is to give, in convenient form, the theoretical bending moments and shearing forces for all the various types of loading in common use, for equal standard spans, and equal standard loads, in the case of

(1) Beams continuous over two equal spans,
(2) " " three "
(3) " " five "

from which the moments and shears may be readily obtained for any number of equal spans and any loads.

The error in assuming that the moments and shears for five spans are the same as with a greater number of spans is very slight, and may be neglected; any spans further removed from the end support than the third span being treated as being similar to the third span. In the same way, also, four spans may be treated as three, though the error is rather greater.

The moments and shears given in the various diagrams 1–54 and tables 2–8, are based upon the assumptions

(1) That the end spans are free at the outer ends, and all spans are equal.
(2) That the moment of inertia of the section is constant.
(3) That the supports are rigid.

In the case of diagrams 55–69 and tables 9–14 the end spans, on the contrary, are assumed fixed and carrying a certain definite bending moment, which is further discussed in Chapter VII.

Although the assumption of constant Moment of Inertia is not entirely justified, the error consequent upon its adoption is generally capable of being ignored.

The cases of loading dealt with are as follows :—

(A) Dead load of structure only.
(B) One point load in centre of span.
(C) Two point loads, dividing the span into three equal parts.
(D) Three ,, ,, ,, four ,,
(E) Four ,, ,, ,, five ,,
(F) Equally distributed load.

In all cases the spans are taken as 100 units and each point load is taken as 100 units, the distributed load being equal to 100 units on each span.

For example, referring to Diagrams Nos. 40–69, the Free Bending Moment for a detached span is expressed as, 1250, 2500, 3333, 5000 or 6000 Units, as the case may be.

Taking first case (F) for equally distributed load,

The Free B.M. is $\frac{w\,L^2}{8}$; and since $w\,L = 100$ and $L = 100$

$$\frac{w\,L^2}{8} = \frac{100 \times 100}{8} = \underline{1250 \text{ Units.}}$$

Also taking case (B) for one point load in the centre of the span,

$$\text{The Free B.M.} = \frac{w\,L}{4} = \frac{100 \times 100}{4} = 2500 \text{ Units.}$$

NOTE.—In all cases the load is taken as covering one or more complete spans.

CHAPTER II.

THE general Theorem of Three Moments may be deduced as follows :—

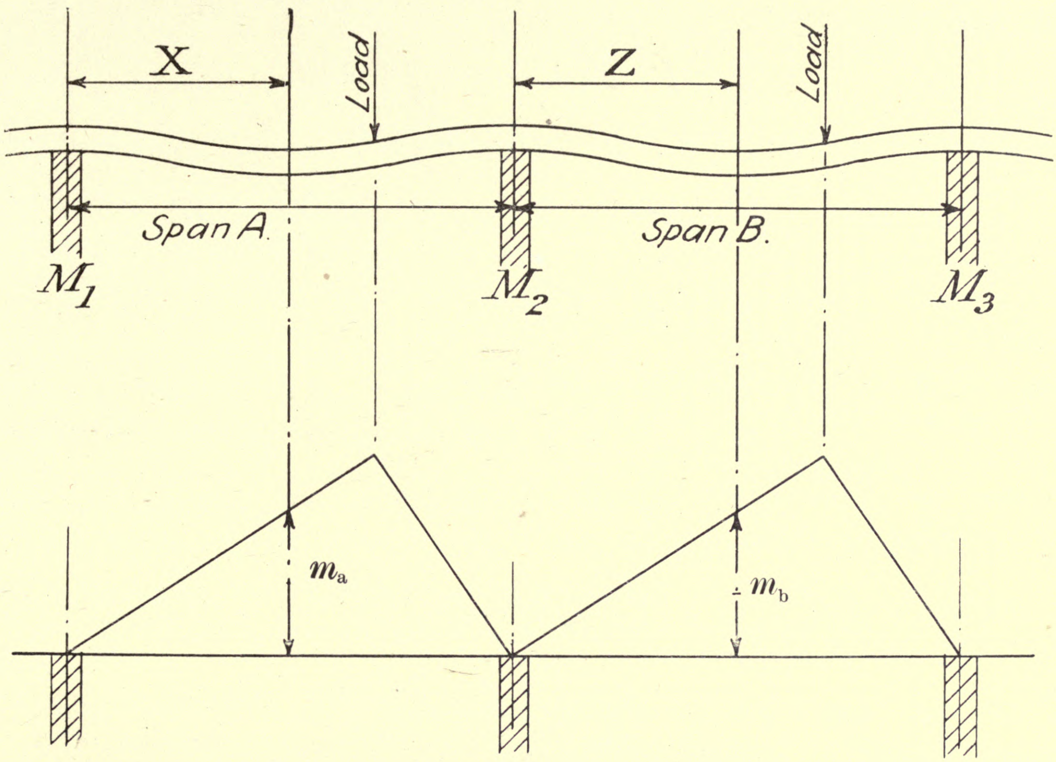


Fig 1.

Taking any two spans, A and B, of a series with any loading, first draw the free bending moment diagrams, and let m_a and m_b be their amounts at distances x and z from the first two piers respectively. (Fig. 1.)

Assuming the beam cut through at the centre support, then the bending moment there is $= 0$, and we have the amount of the negative moment at $x = M_1 \frac{A - X}{A} - m_a$ and at $z = M_3 \frac{Z}{B} - m_b$.

Now, if we assume the girder joined again, since M_1 and M_3 are fixed by side spans, and m_a and m_b do not depend upon the bending of the girder, the only variable is M_2, which by the 'principle of least work' must be such that the work done in bending the beam is a minimum, and therefore if the work done $= u$, then $\frac{d u}{d M_2}$ must be $= 0$.

Let the total negative moments at x and z = M_a and M_b respectively, then :—

$$M_a = M_1 \left(\frac{A - x}{A}\right) + M_2 \frac{x}{A} - m_a$$

and

$$M_b = M_3 \frac{z}{B} + M_2 \left(\frac{B - z}{B}\right) - m_b$$

The work done in bending the beam $= \frac{1}{2}\int \frac{M_2}{E\,I} (d\,x)$ and the work done in the case under consideration equals

$$\frac{1}{2\,E}\left\{\int_0^A \frac{(M_a)^2}{I} (d\,x) + \int_0^B \frac{(M_b)^2}{I} (d\,z)\right\}$$

Differentiating with respect to M_2, and equating with zero, and multiplying by E, we have

$$\int_0^A \left(M_1 \frac{A - x}{A} + M_2 \frac{x}{A} - m_a\right) \frac{x\,(d\,x)}{A\,I}$$

$$= - \left\{\int_0^B \left(M_2 \frac{B - z}{B} + M_3 \frac{z}{B} - m_b\right) \frac{B - z\,(d\,z)}{B\;\;I}\right\} *$$

which is the general expression from which, assuming E to be uniform, any case can be solved.

If, however, as is generally the case, the girder is (for all practical purposes) of uniform section, I is constant and may be omitted.

Then, by integrating the expressions containing the pier moments, and bringing those affecting the free bending moment diagrams to the other side, we have, after reducing

$$M_1\,A + 2\,M_2\,(A + B) + M_3\,B = 6\left(\frac{\int_0^A m_a\;x\;(d\;x)}{A} + \frac{\int_0^B m_b\,(B - z)\,(d\;z)}{B}\right)$$

$$= 6\left(\frac{S_1\,Y_1}{A} + \frac{S_2\,Y_2}{B}\right) \dagger \quad \text{See Fig. 2.}$$

This expression is of the simplest possible form, and is based upon the assumptions stated. See also Fig. 2.

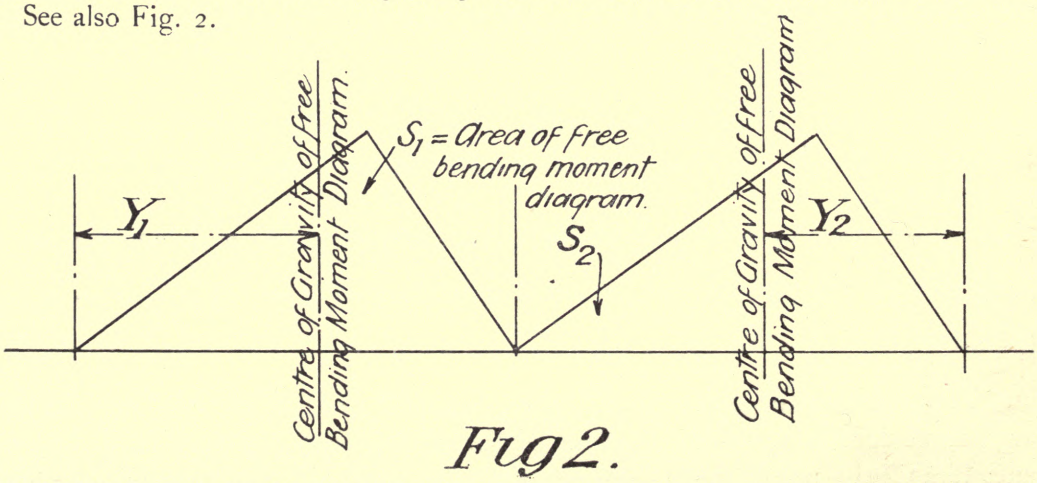


Fig 2.

* For detailed proof see Appendix I.

† If the reader does not wish to follow the reasoning, this result may be taken at once, if I be assumed uniform; I signifying the Moment of Inertia of the section, and E the Modulus of Elasticity.

CHAPTER III.

APART from the question of the fixedness of the outside spans where the beams are attached to columns, to which reference is made later on in Chapter VII., the question of the calculation of the bending moments and shearing forces must of necessity be divided into two parts, one dealing with the 'live' or 'incidental' loads, and the other with the dead loads, or the weight of the structure affecting the beams in question.

The following diagrams, 1–39, show all the various possible arrangements of assumed live loads and for convenience are arranged in order, as follows :—

Diagrams	1–2	show	two	spans, for which	there are	two	possible	cases of loading.
,,	3–7	,,	three	,,	,,	five	,,	,, ,,
,,	8–39	,,	five	,,	,,	eighteen	,,	,, ,,

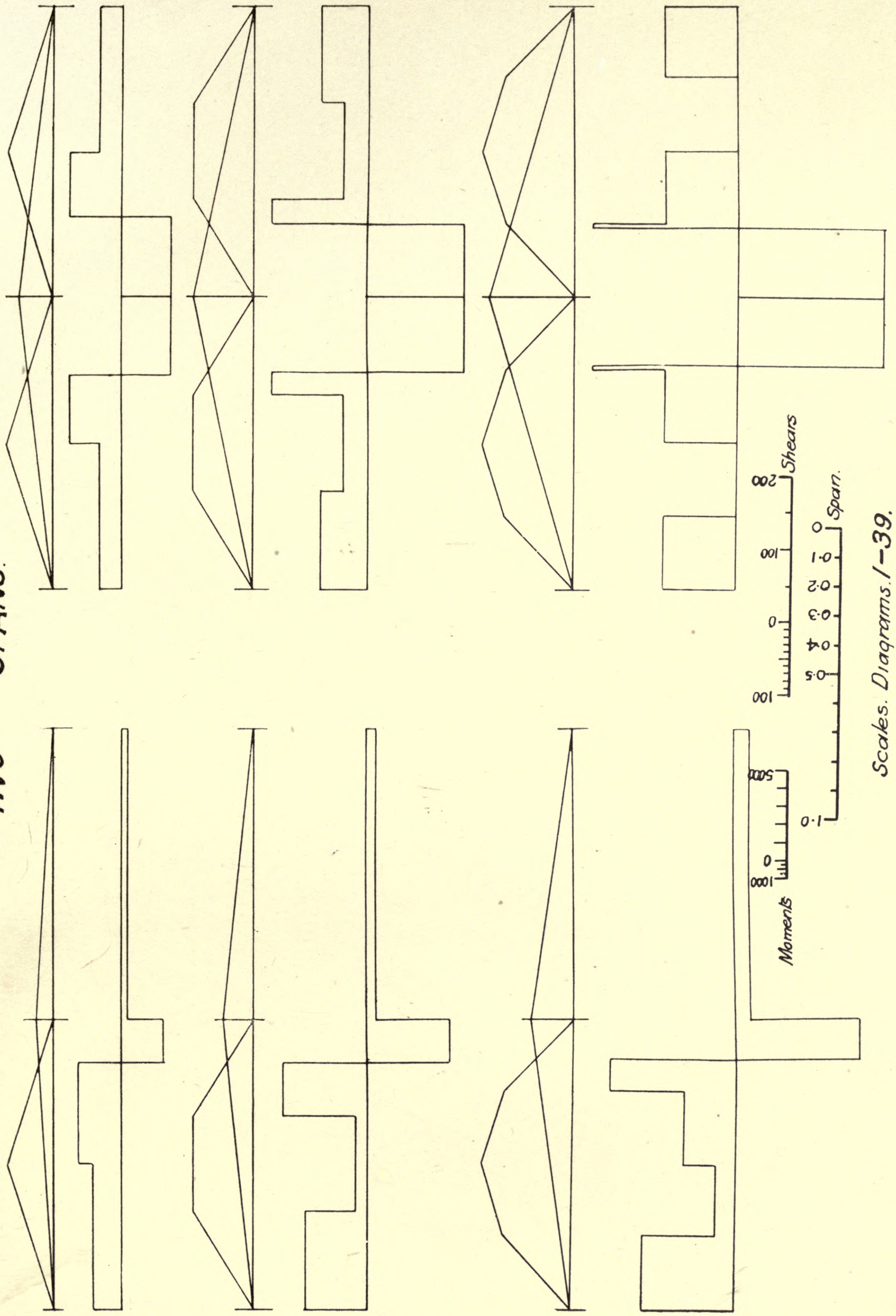
Throughout the diagrams, the loads are taken in the order of Case B, C, D, E, and F.

The dead load diagrams (Case A) are, of course, shown in diagrams of distributed load with all spans loaded.

The difference between the dead loads and distributed loads, in Tables 1–12, is that the dead loads are taken from the one condition of loading only, *i.e.*, distributed load on all spans at one time, and the distributed loads are taken from the entire number of possible arrangements of assumed distributed load with the spans in question, so as to give the maximum possible limiting bending moments and shearing forces, both positive and negative, for the loads assumed.

DIAGRAM No I.

TWO SPANS.



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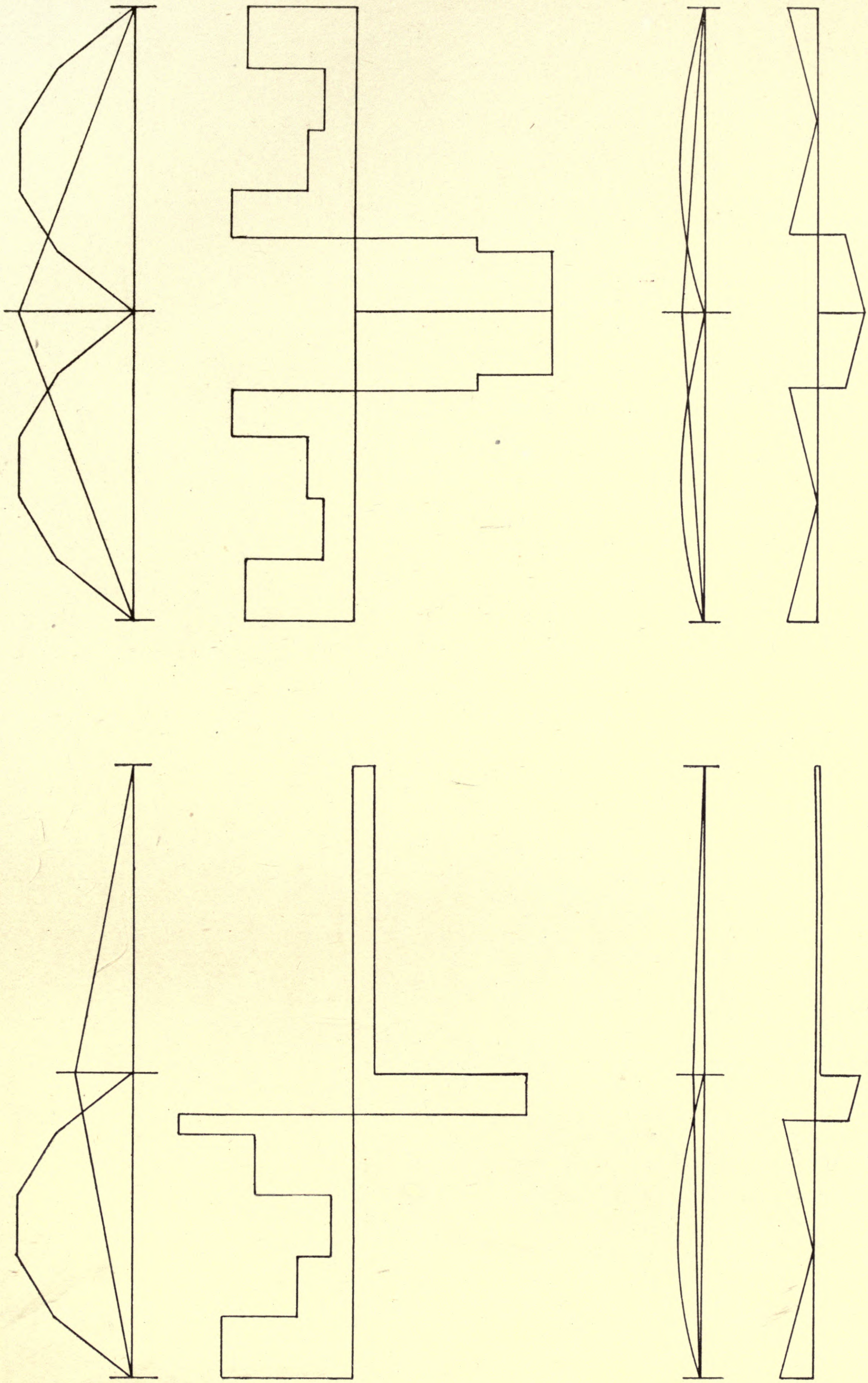
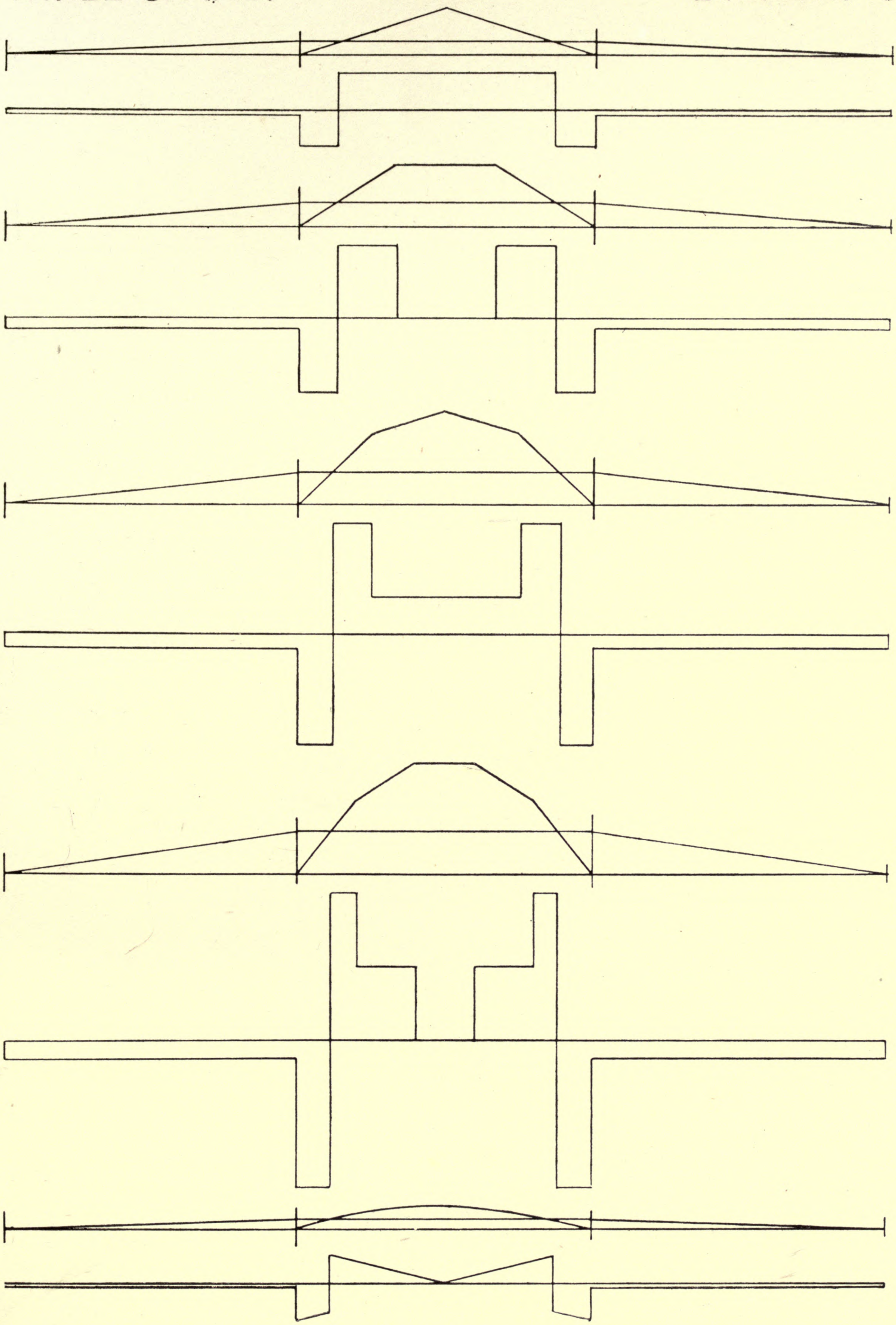


THREE SPANS.

DIAGRAM Nº 3.



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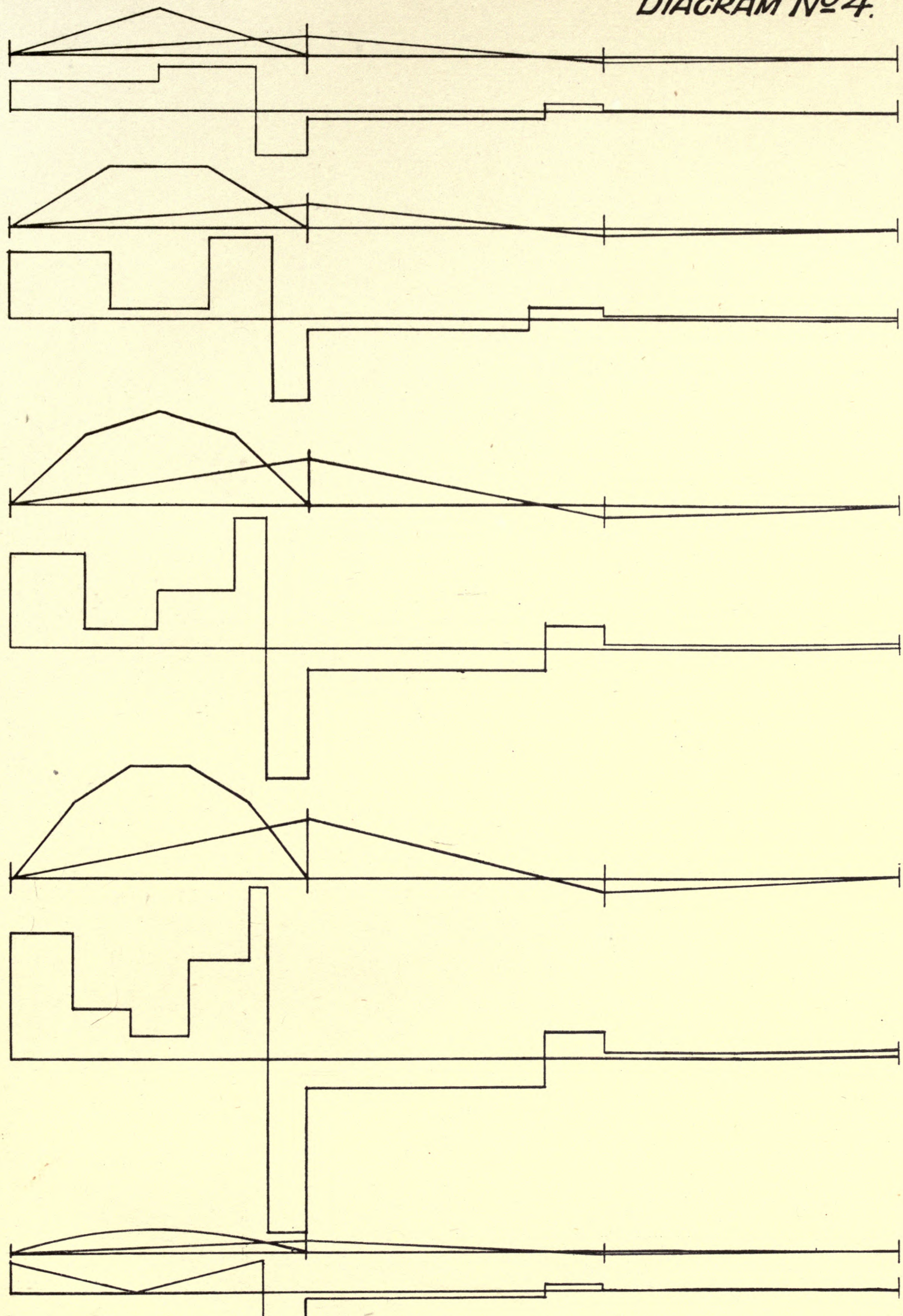

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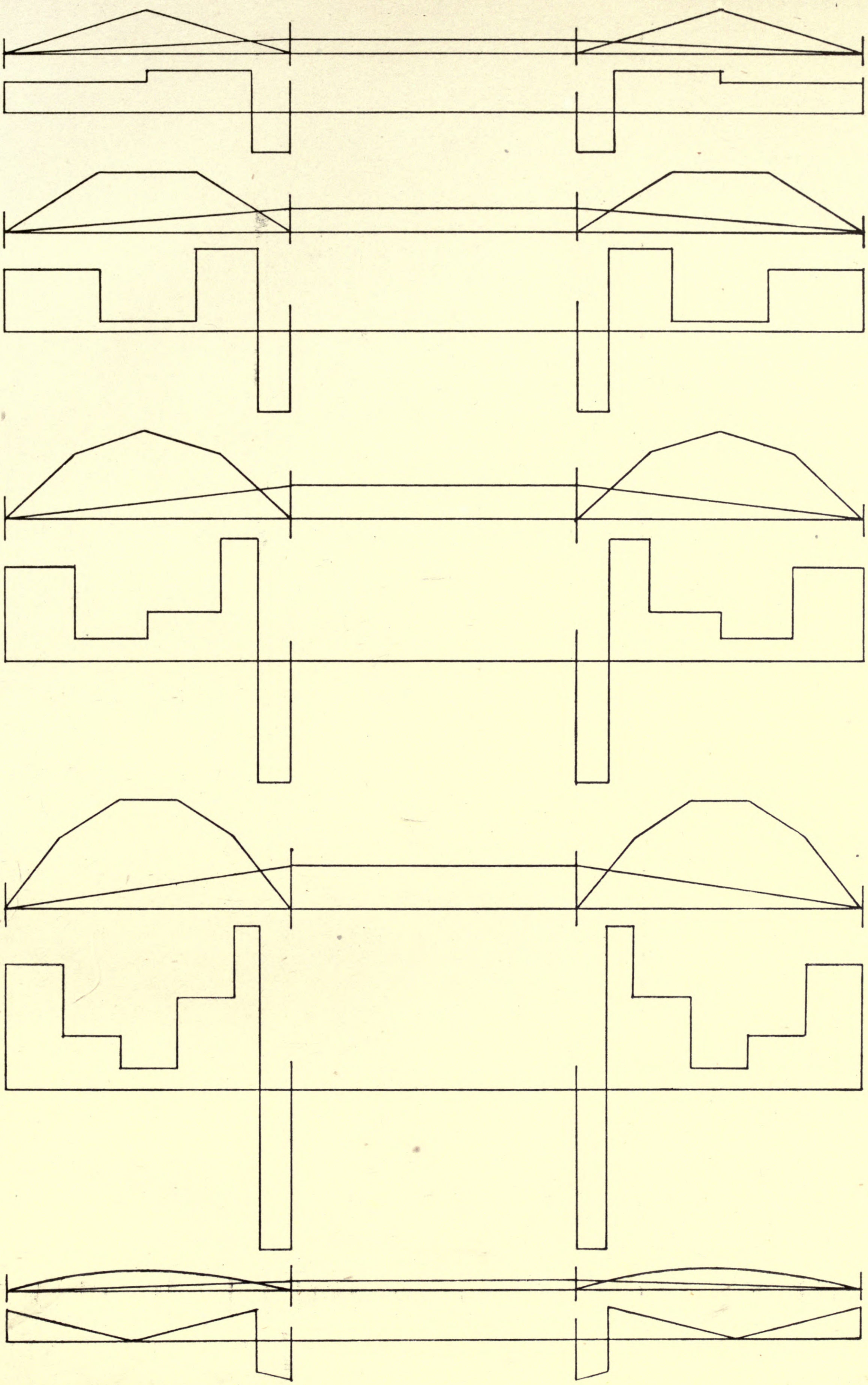

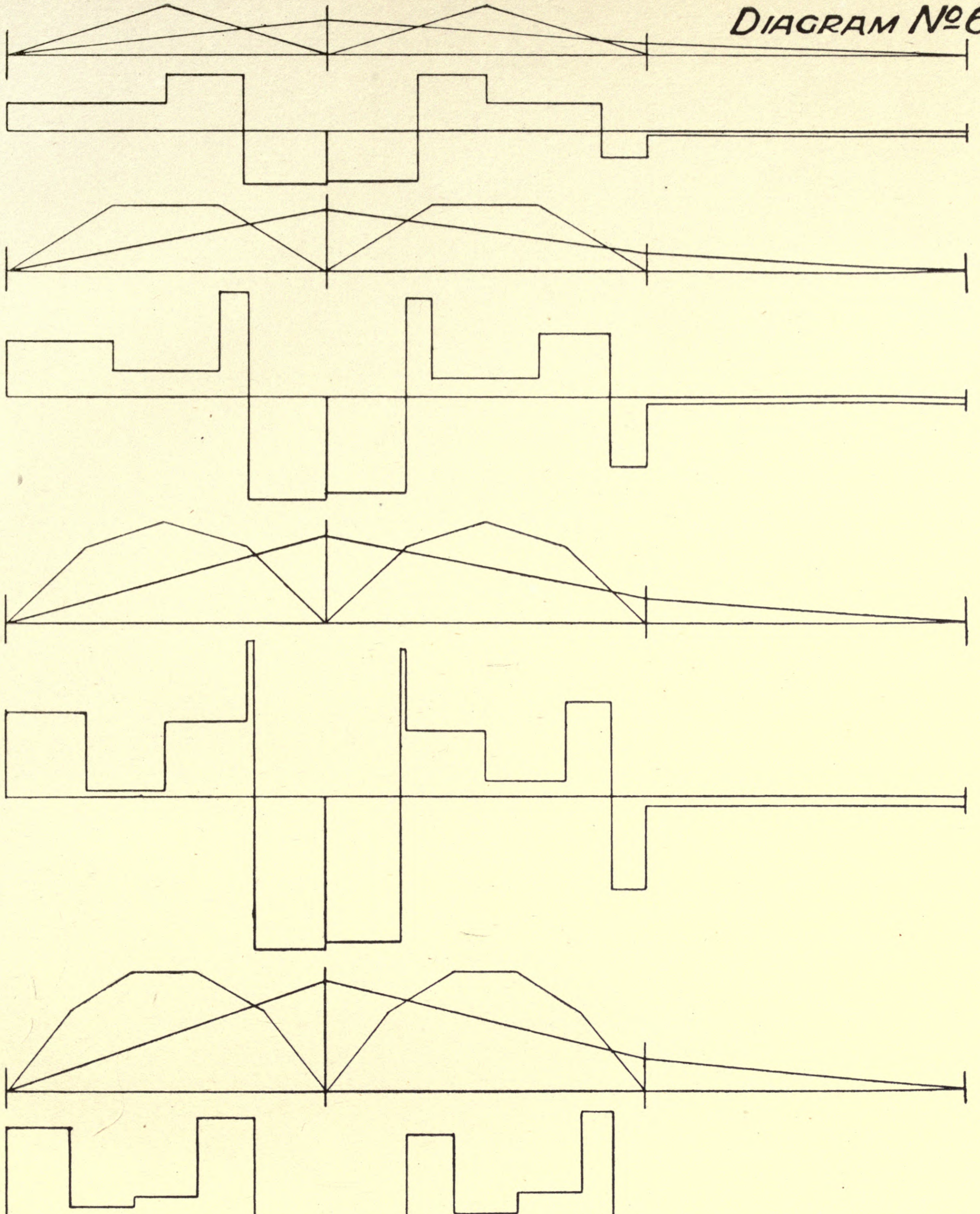
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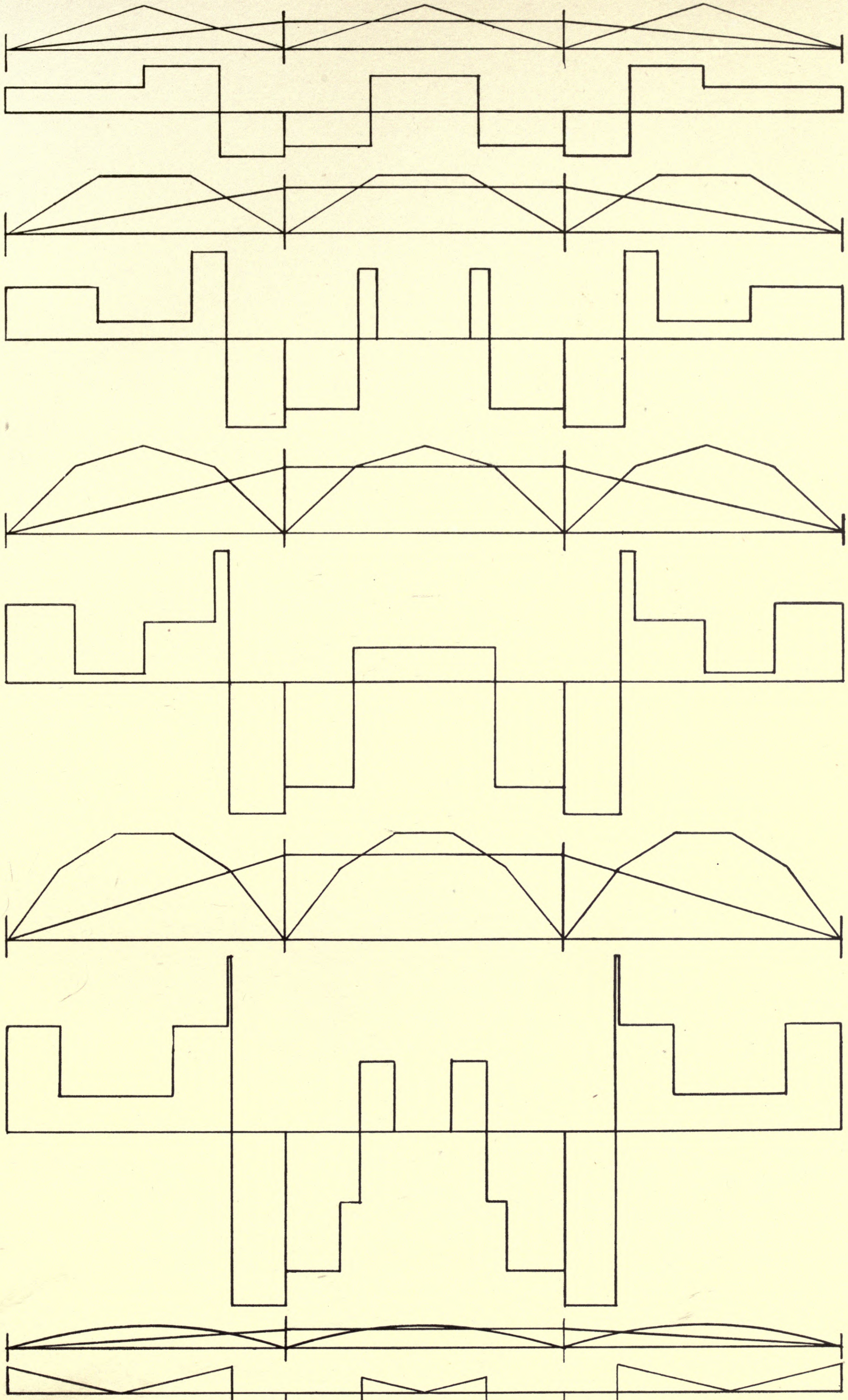
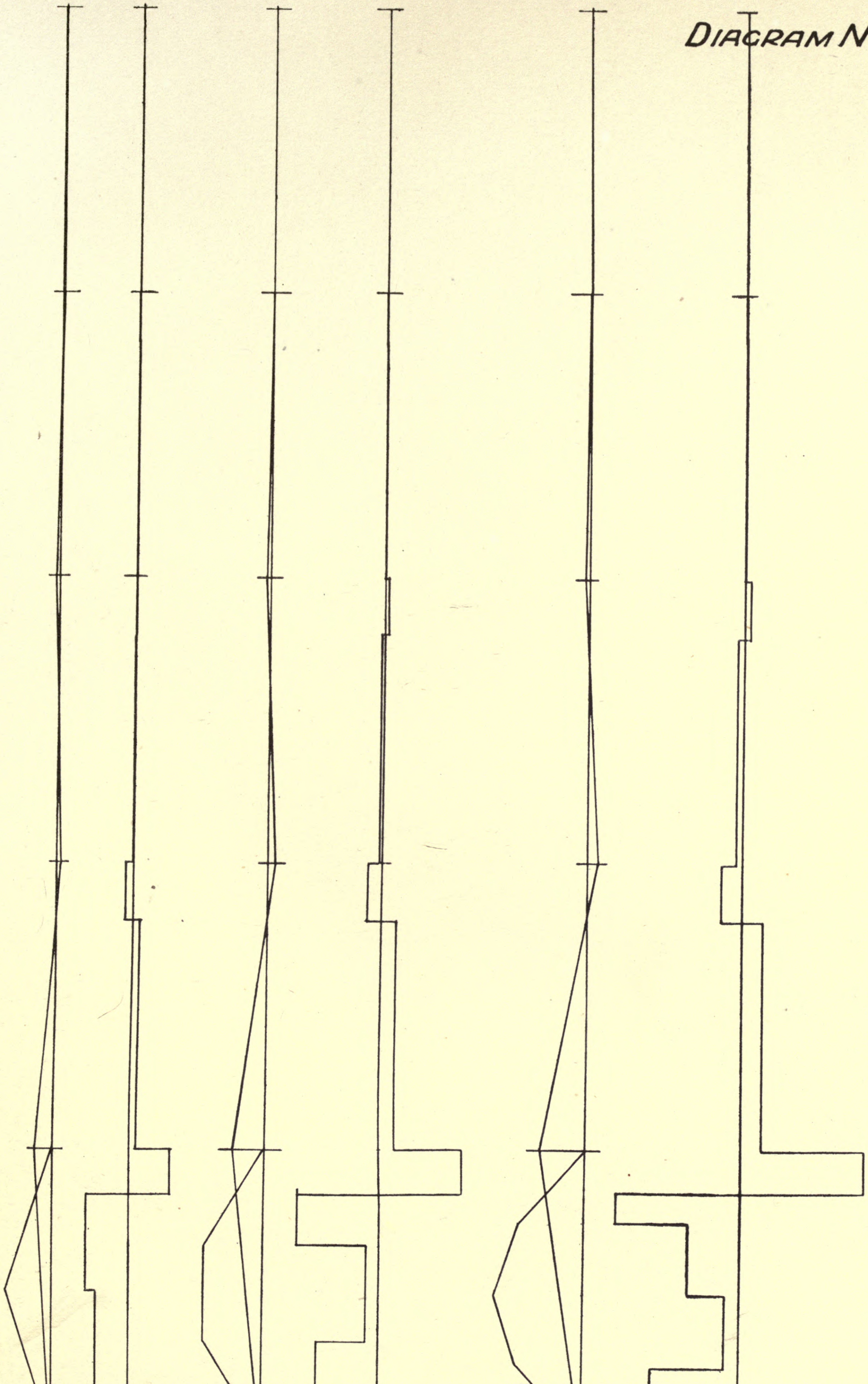
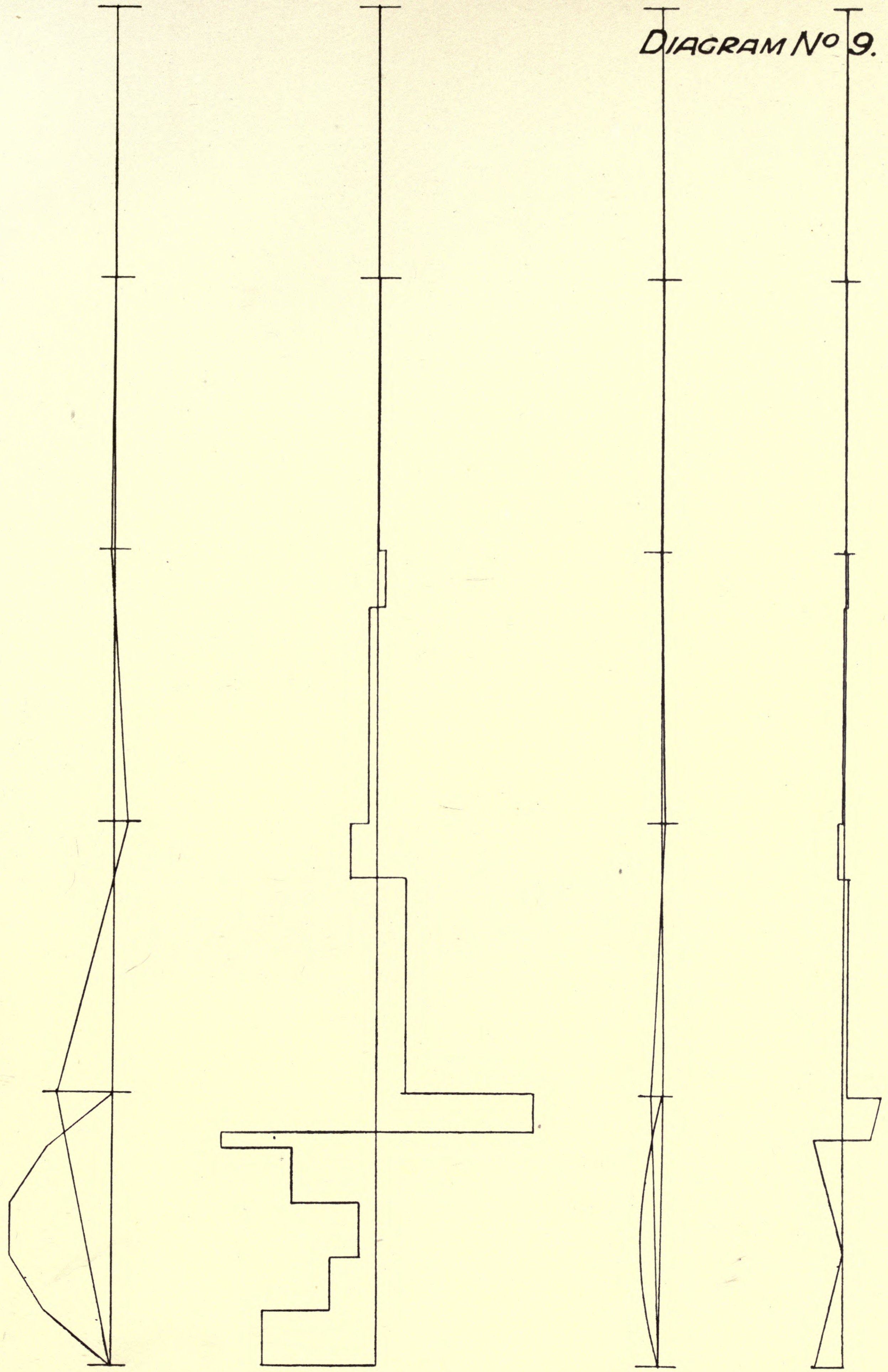


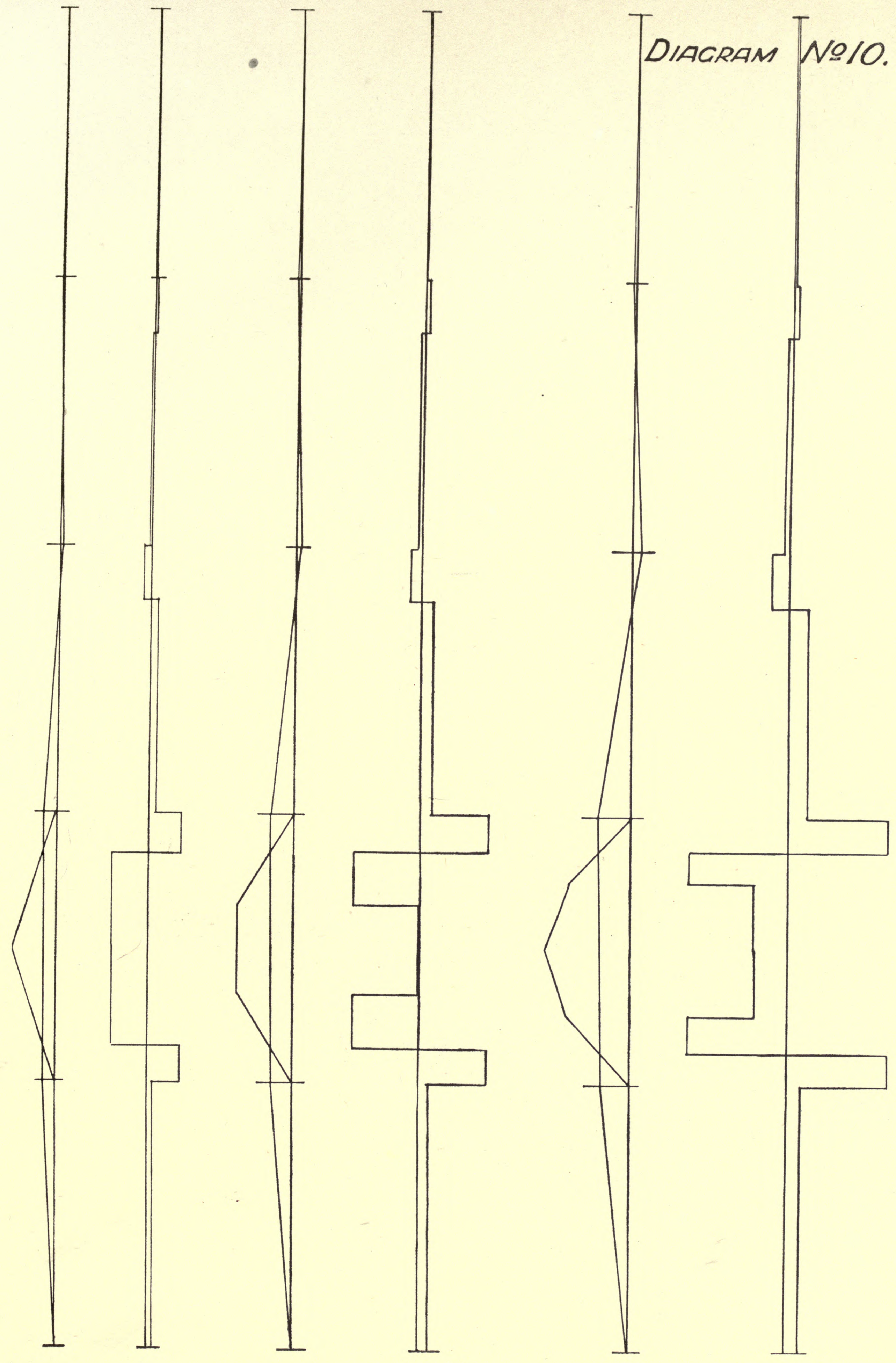
Diagram No. 8.



Five Spans.

DIAGRAM No. 9.

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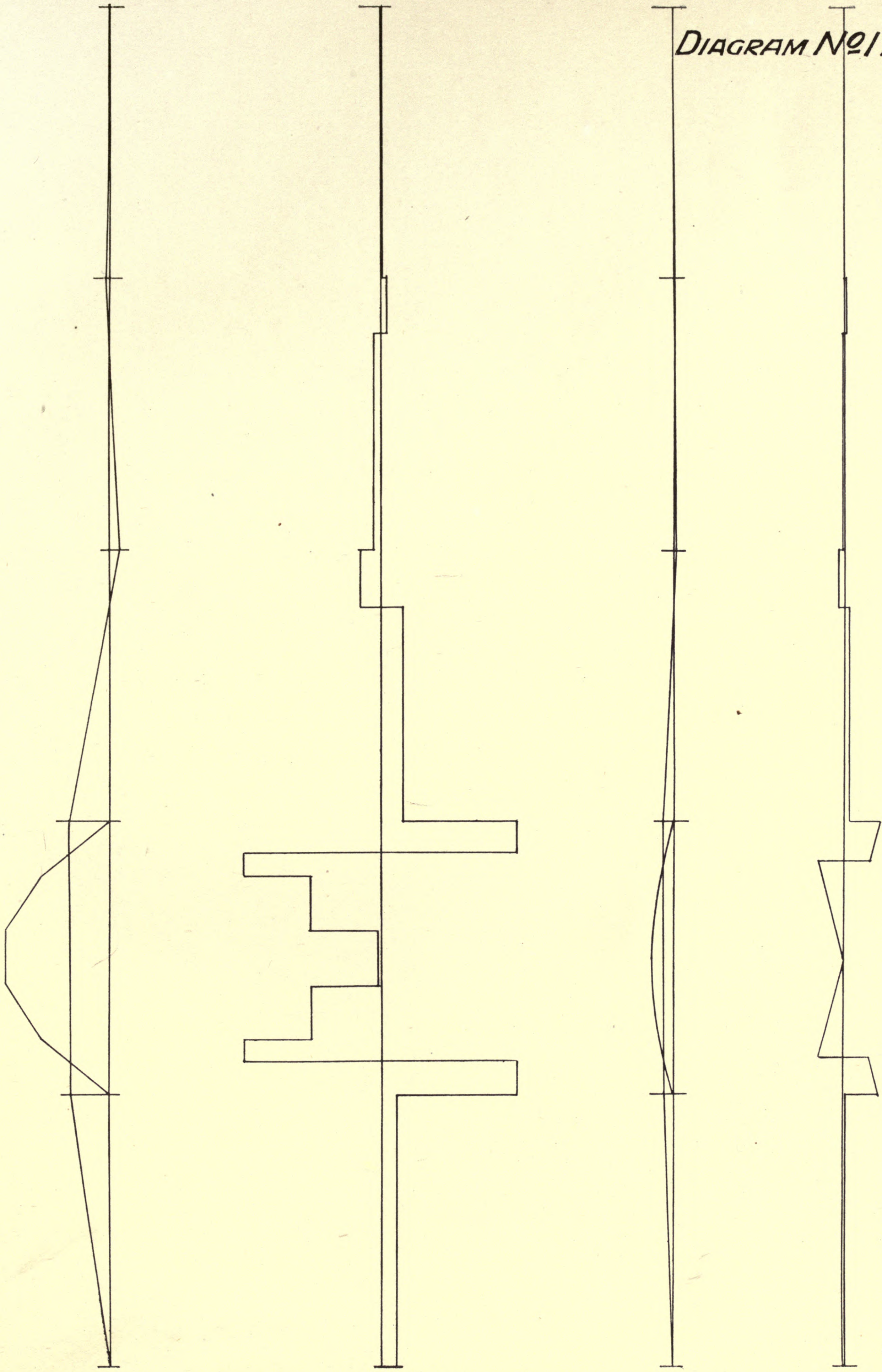

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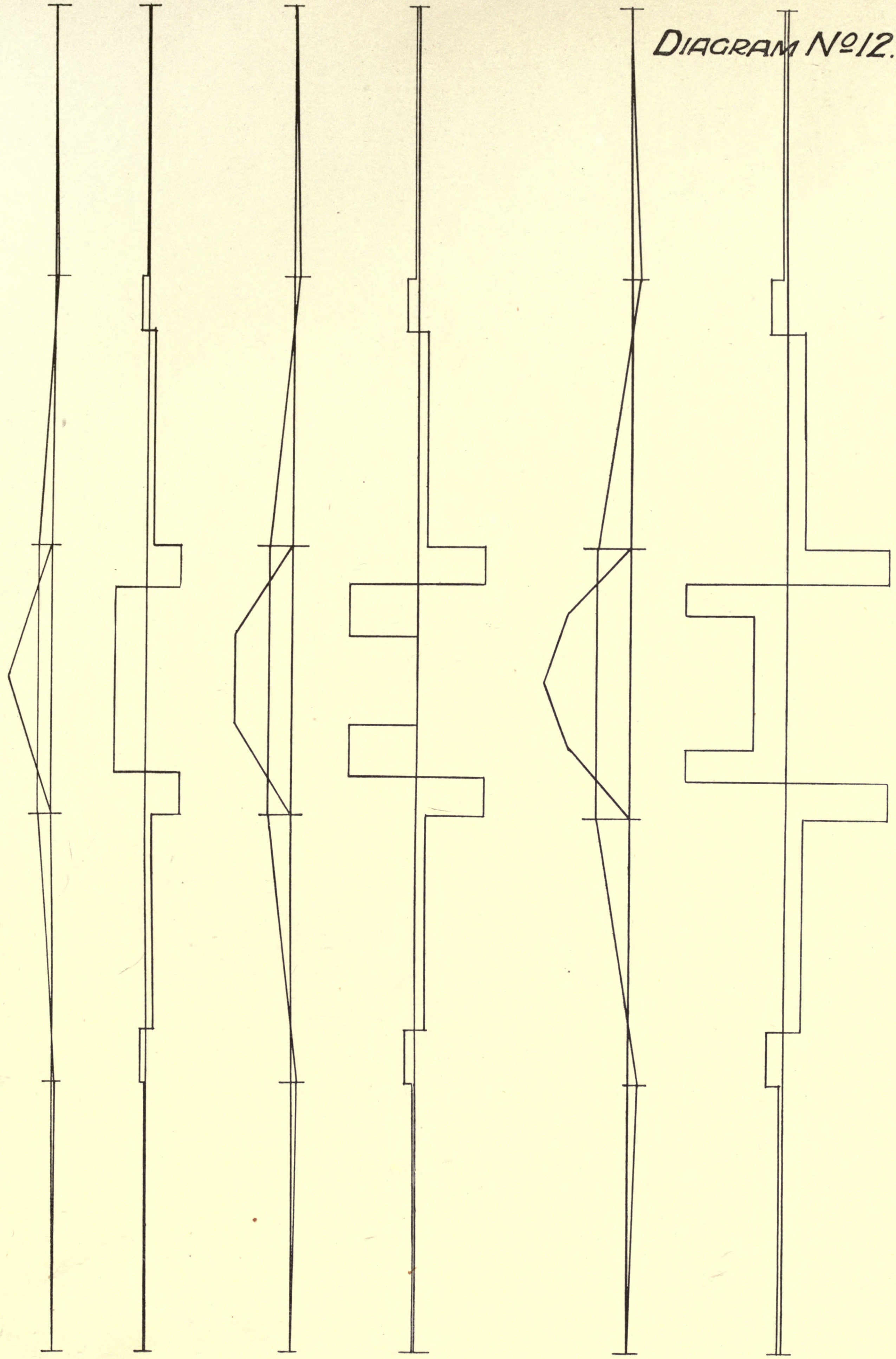

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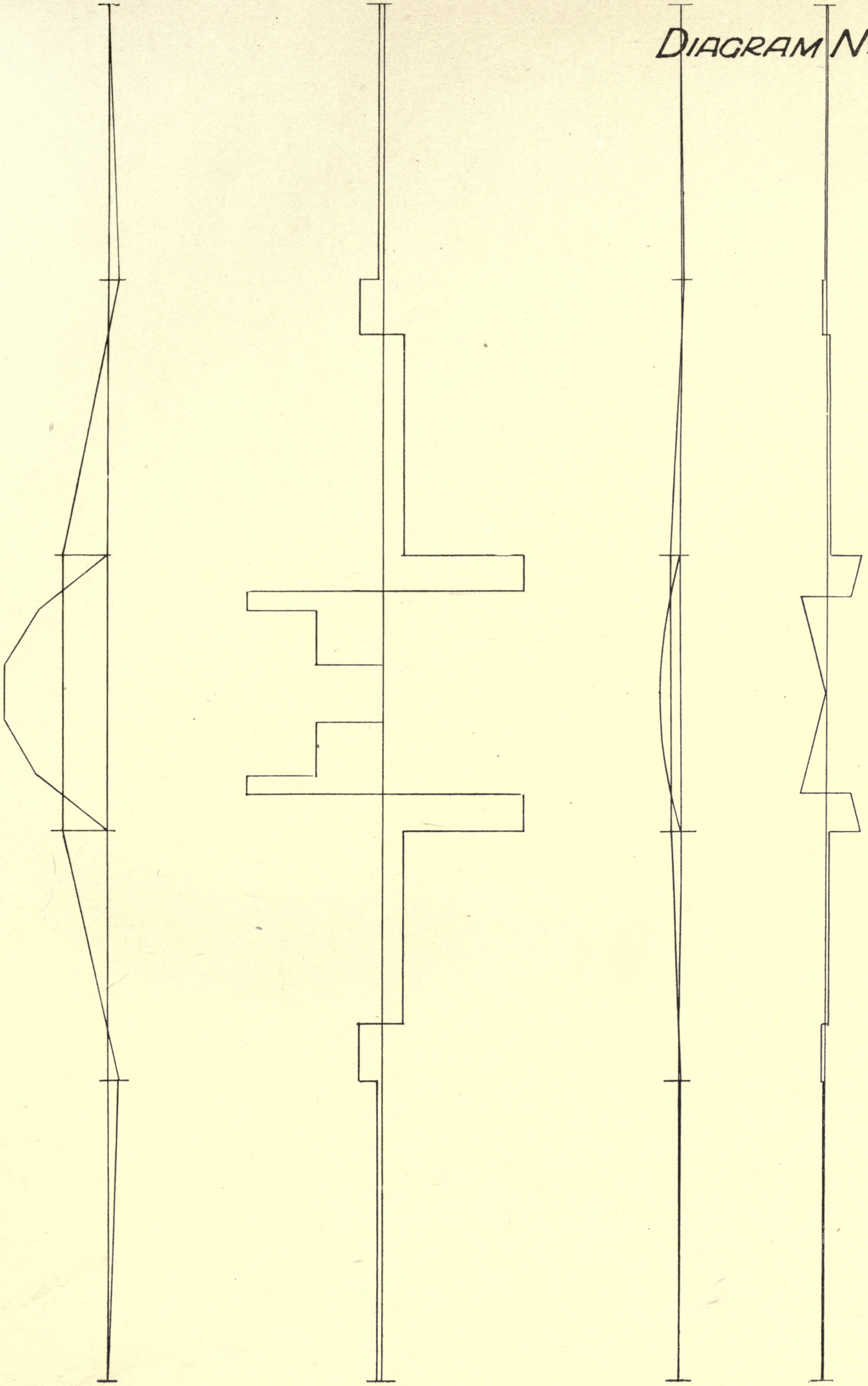

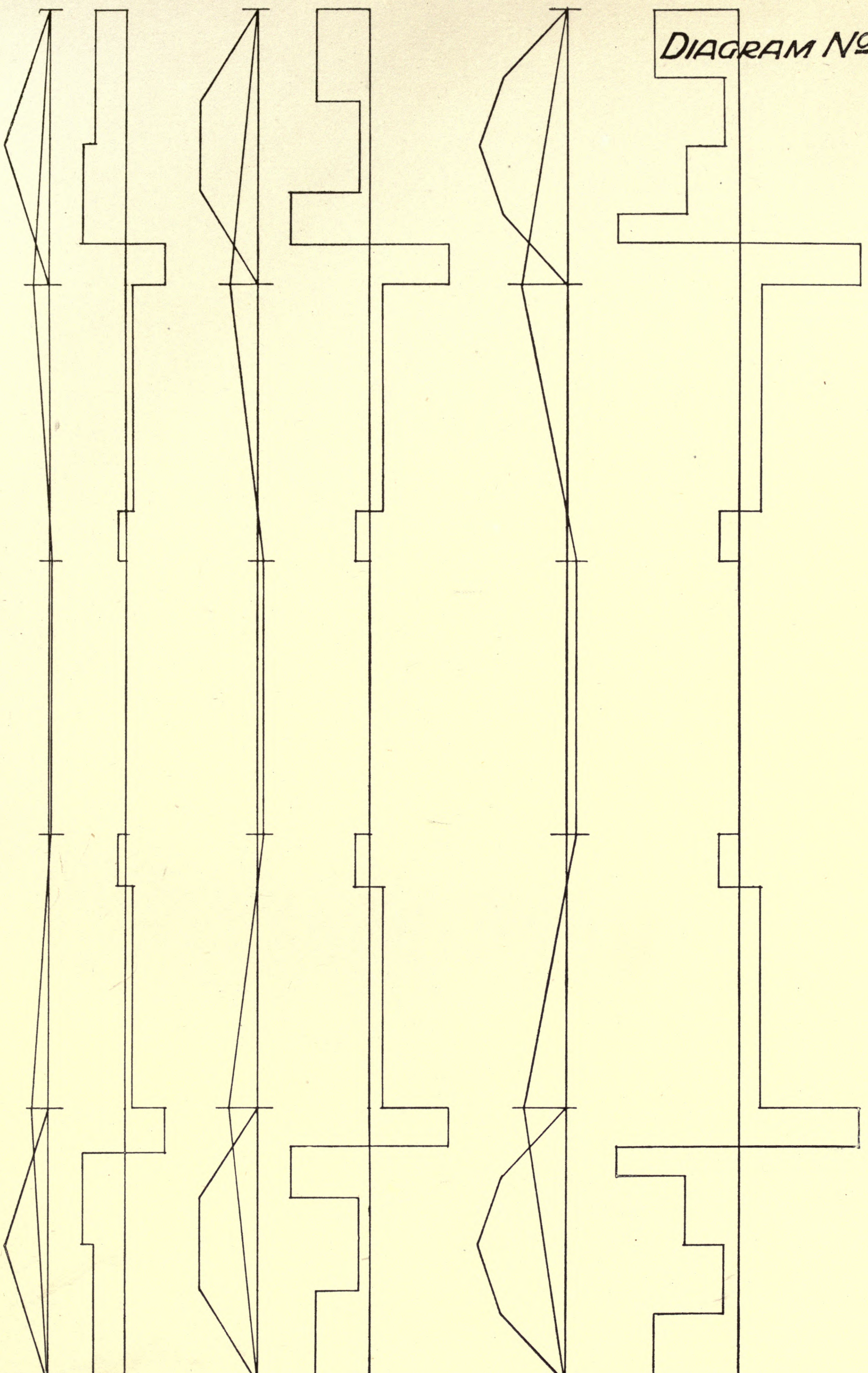

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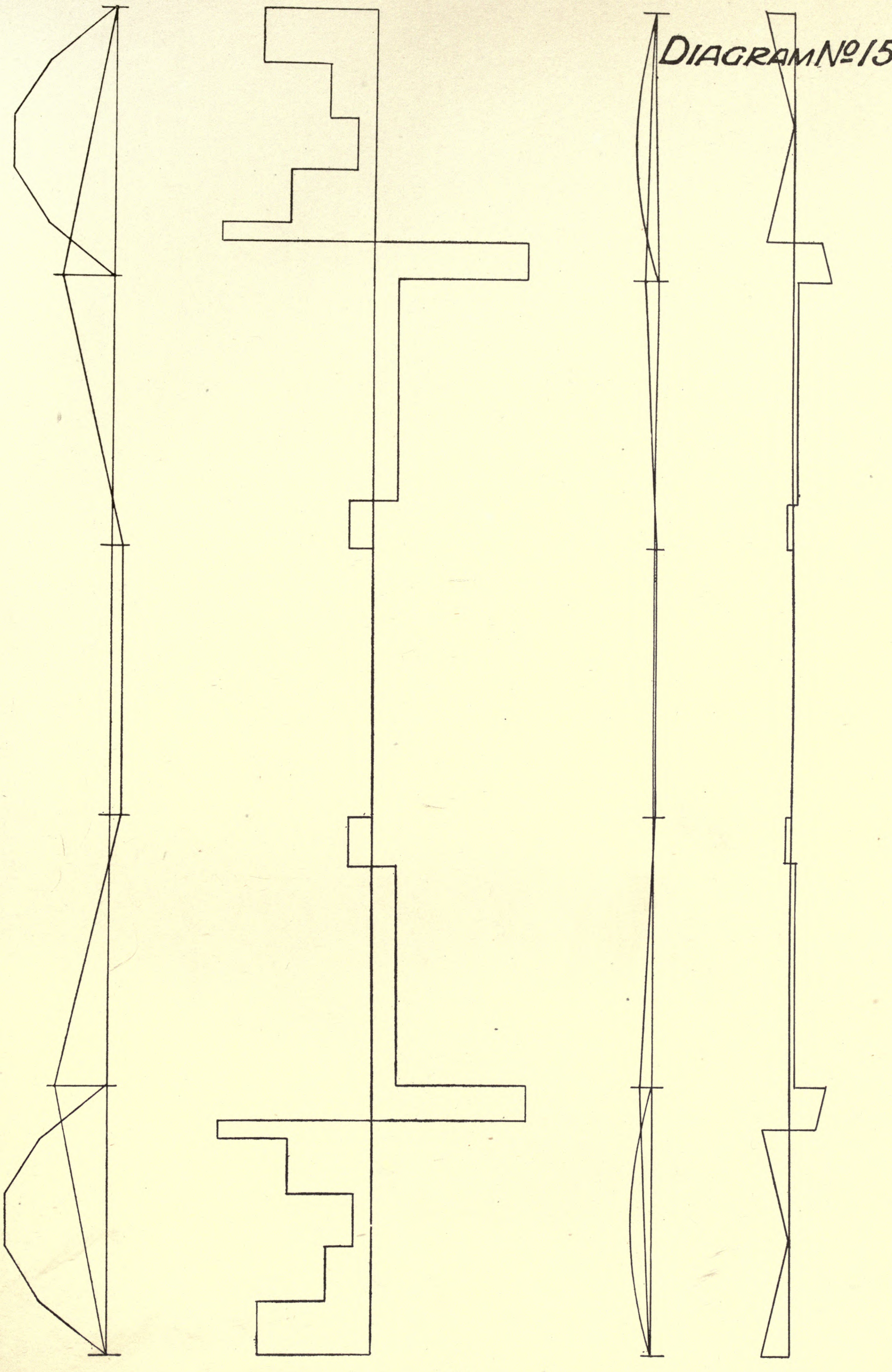
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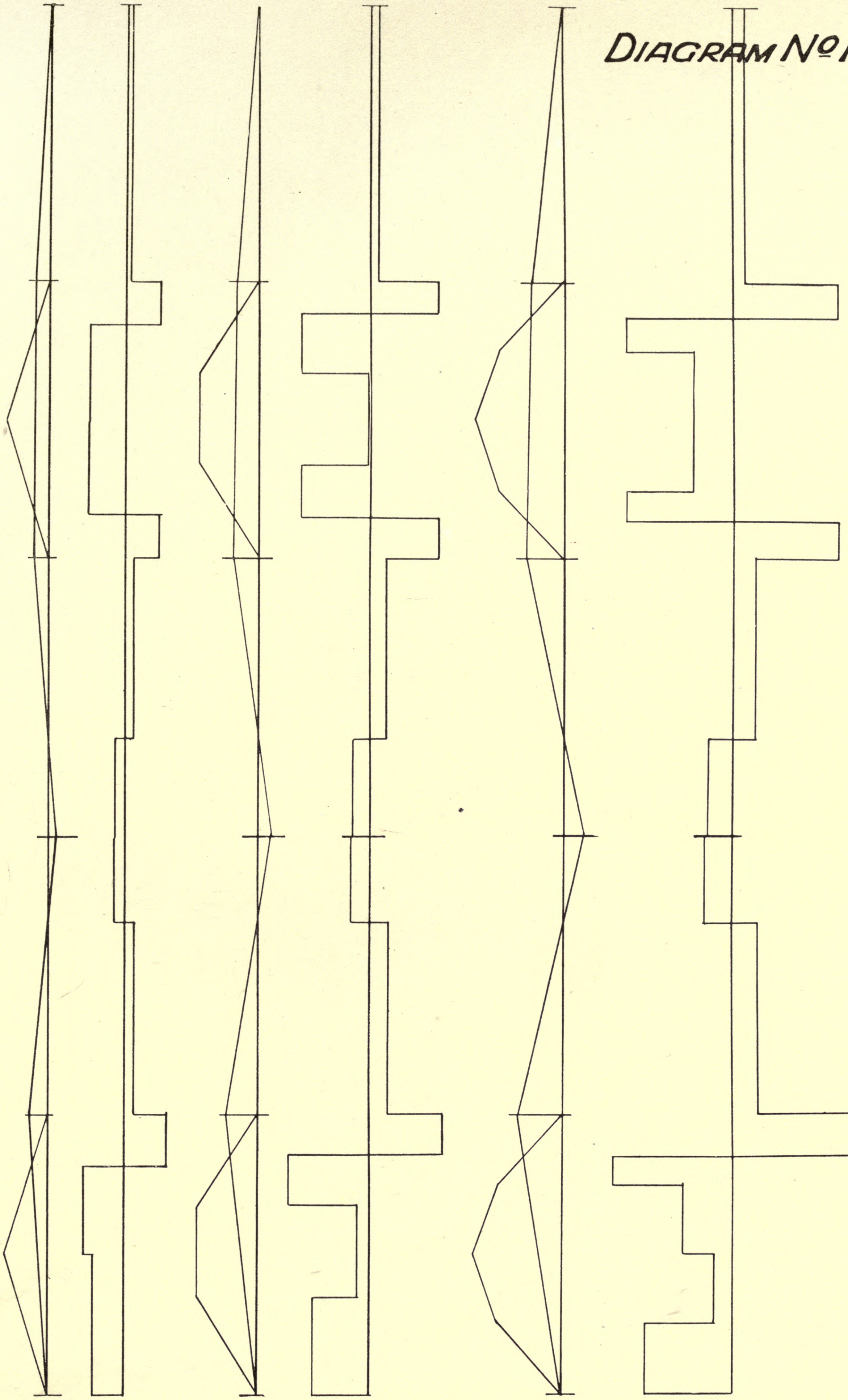

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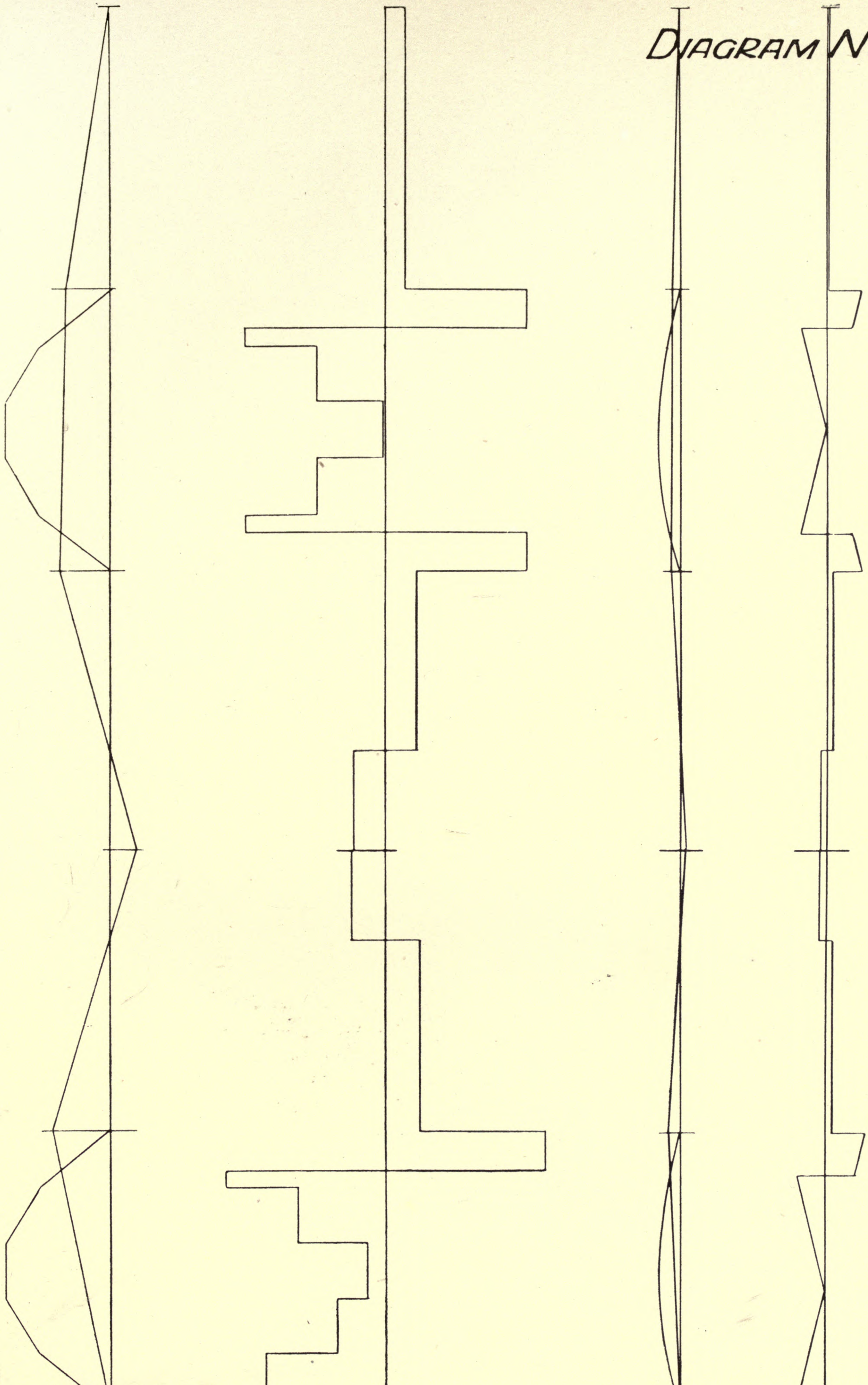

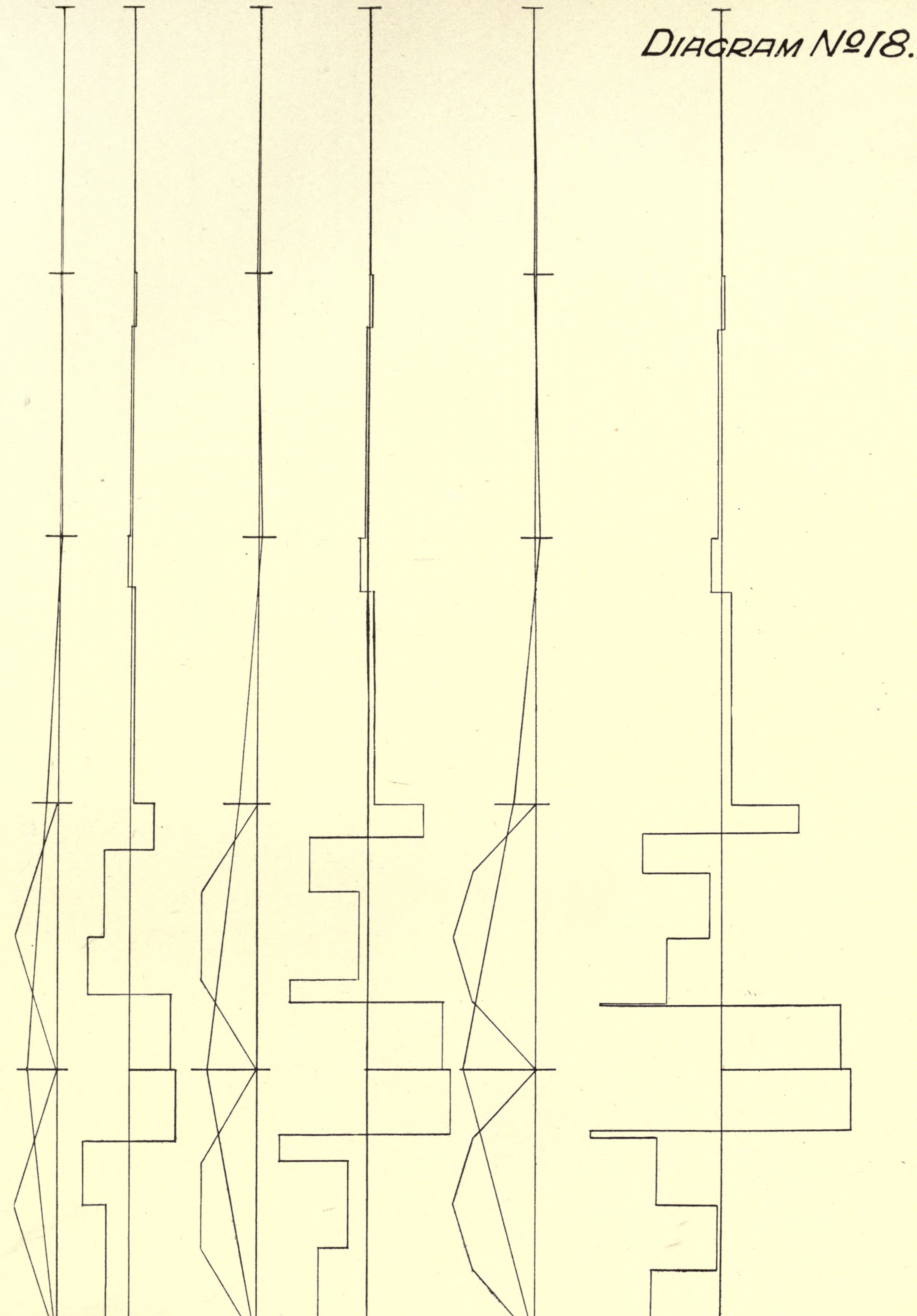
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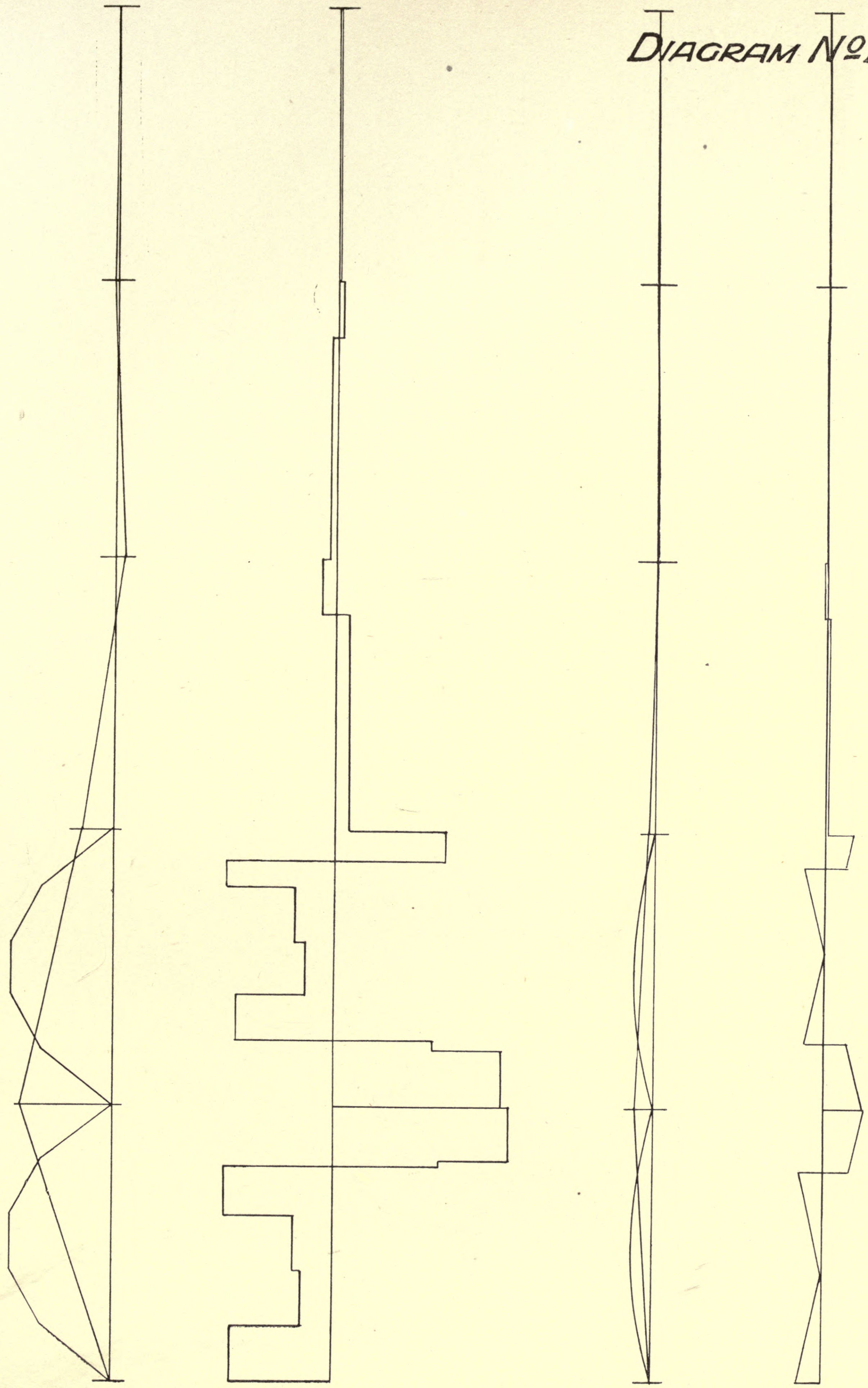

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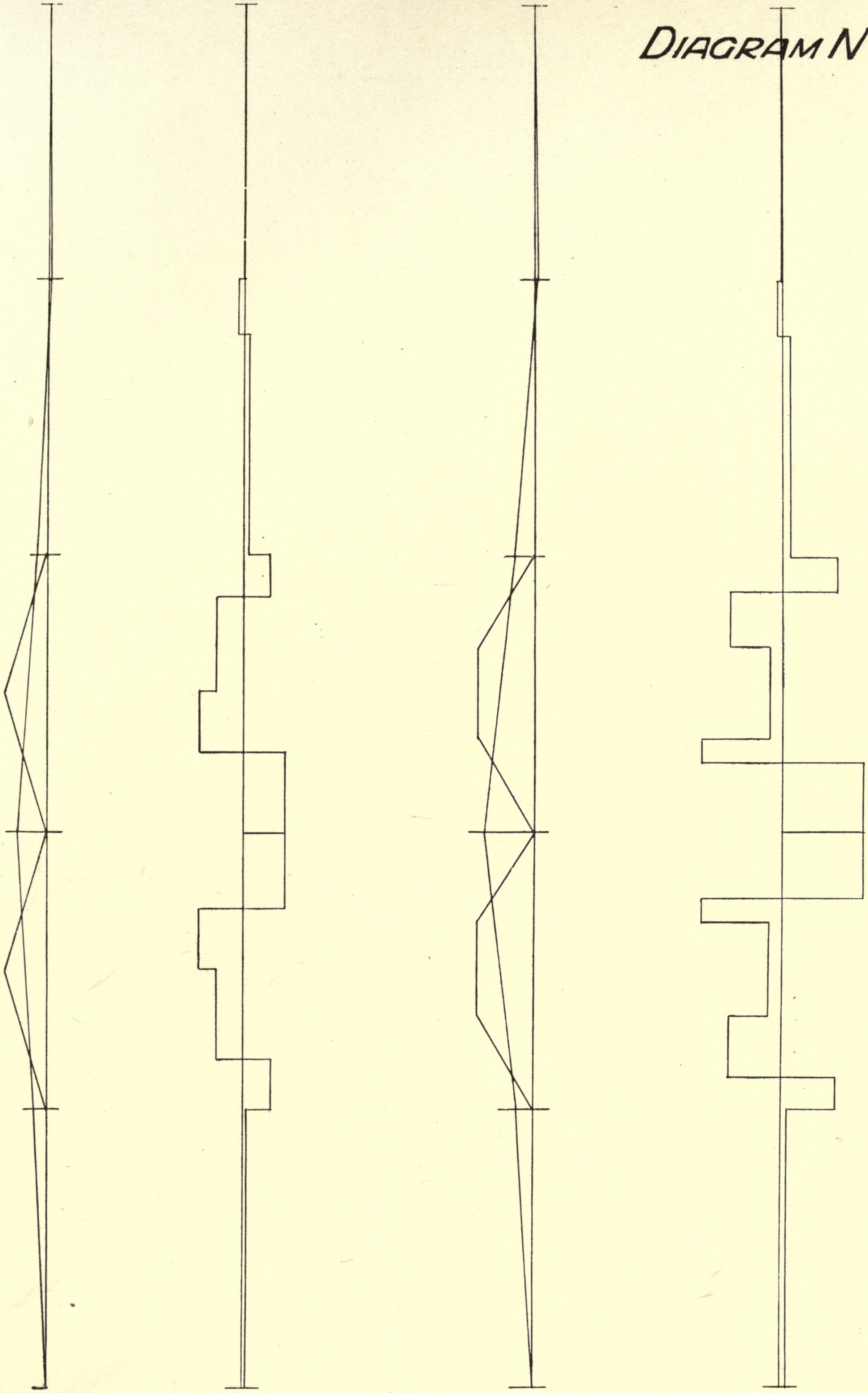

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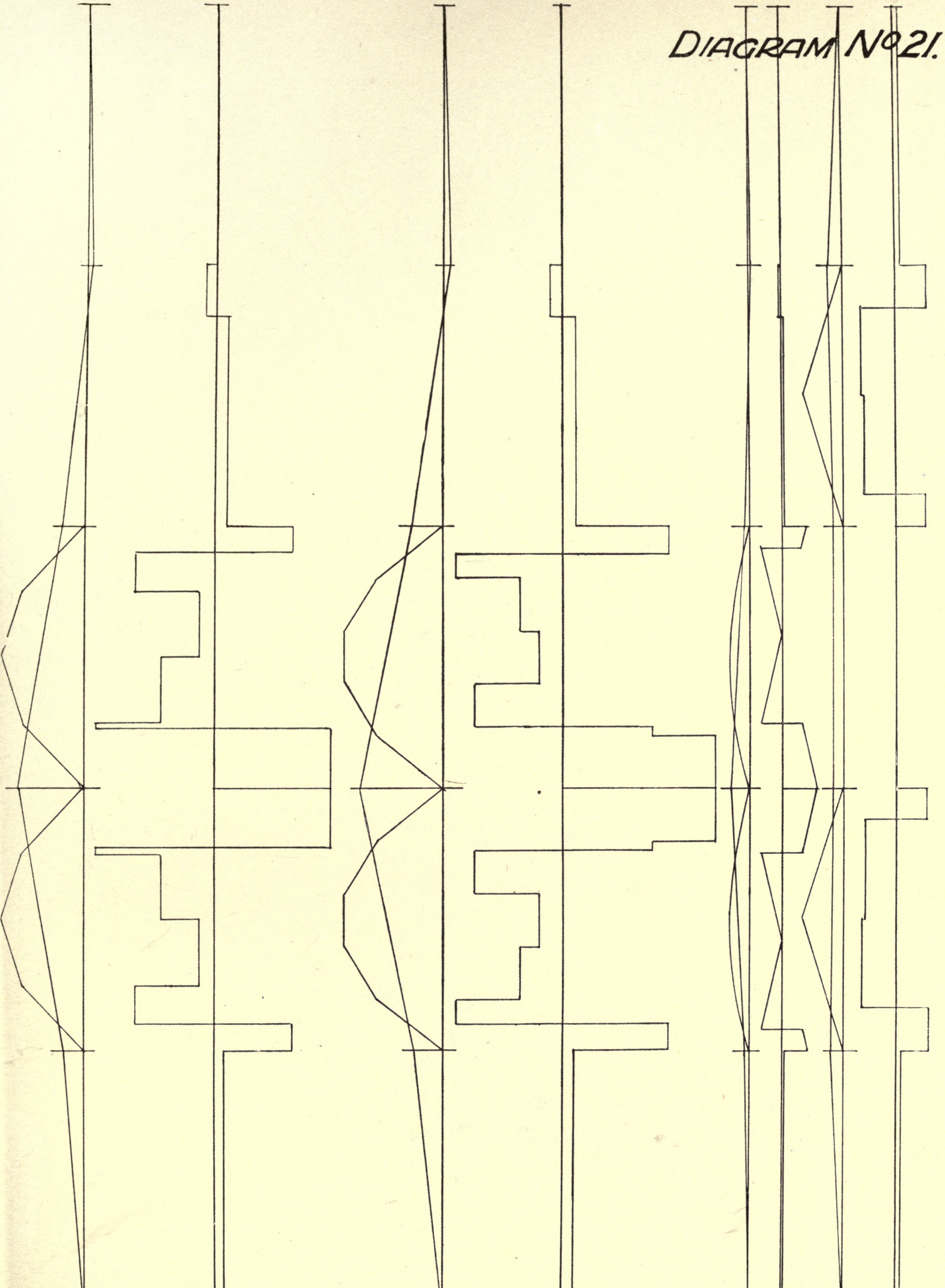

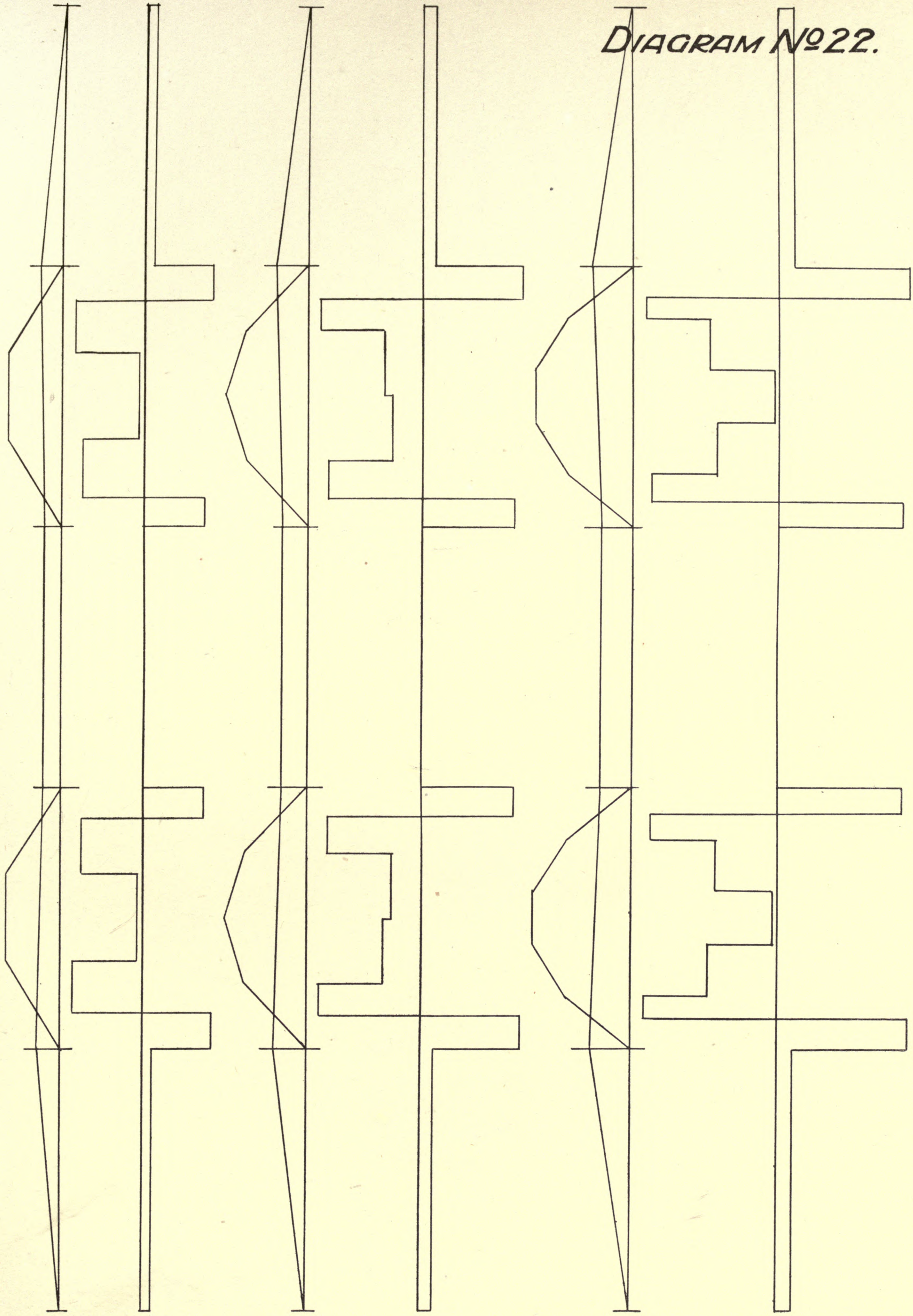
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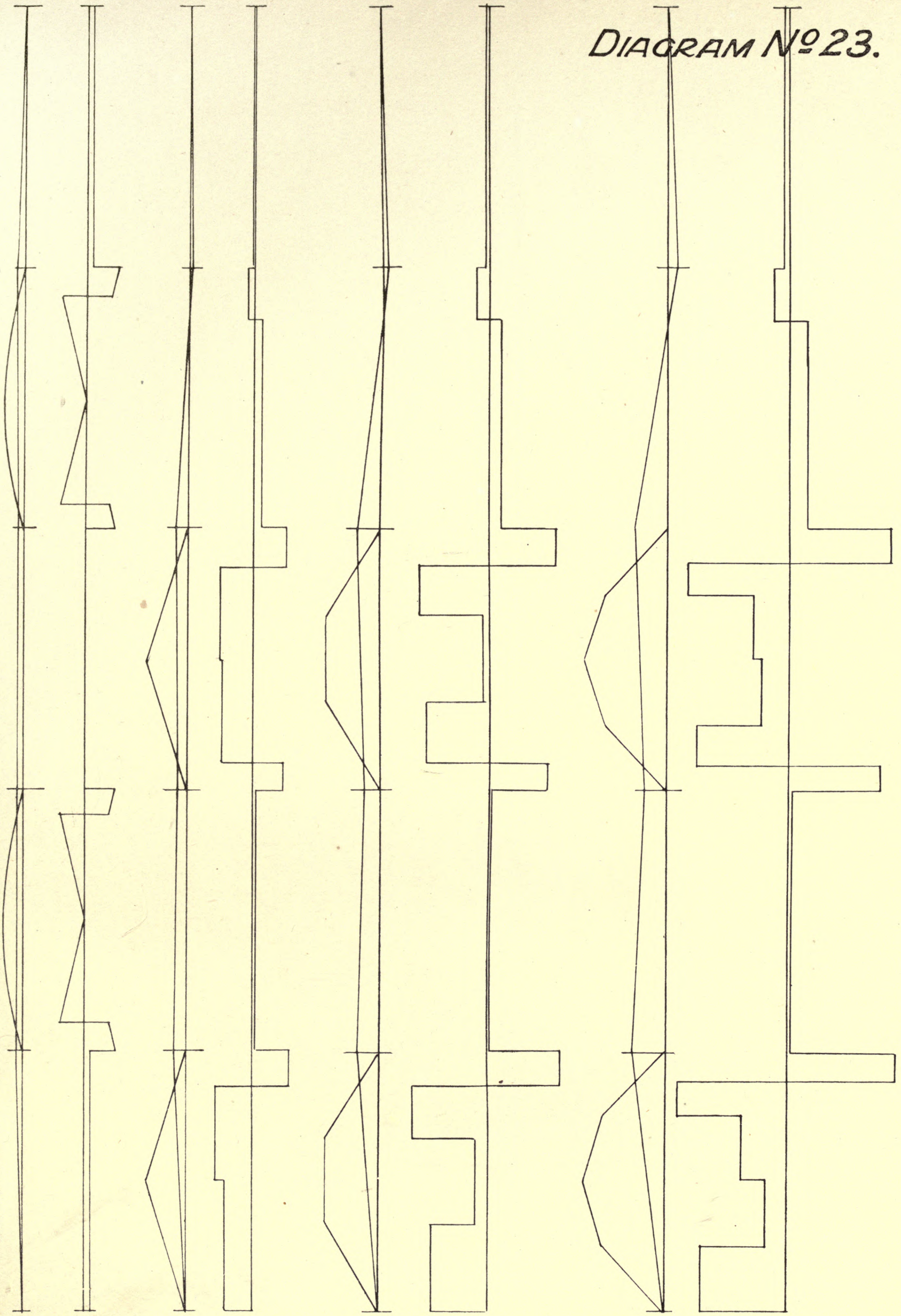

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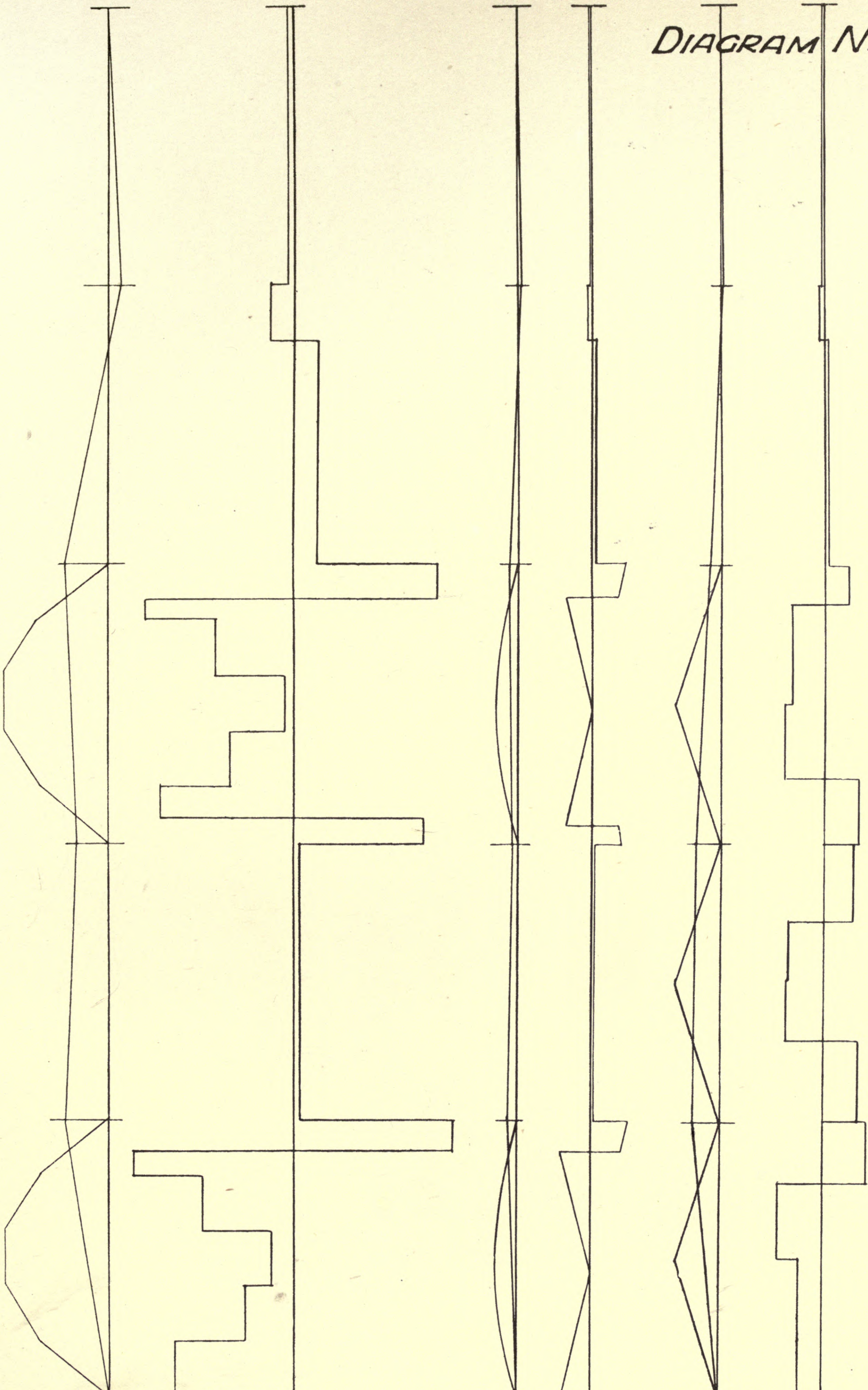

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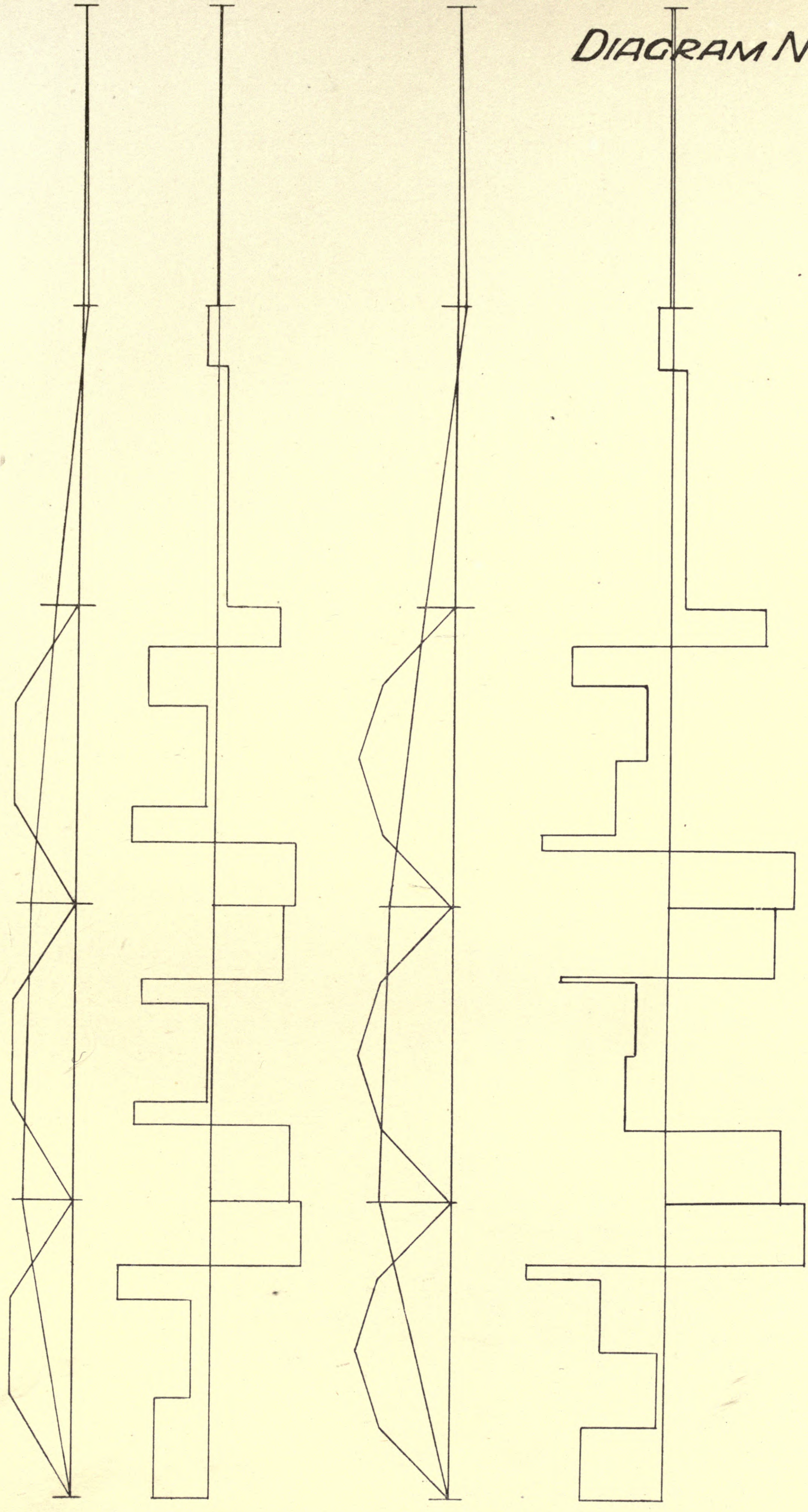

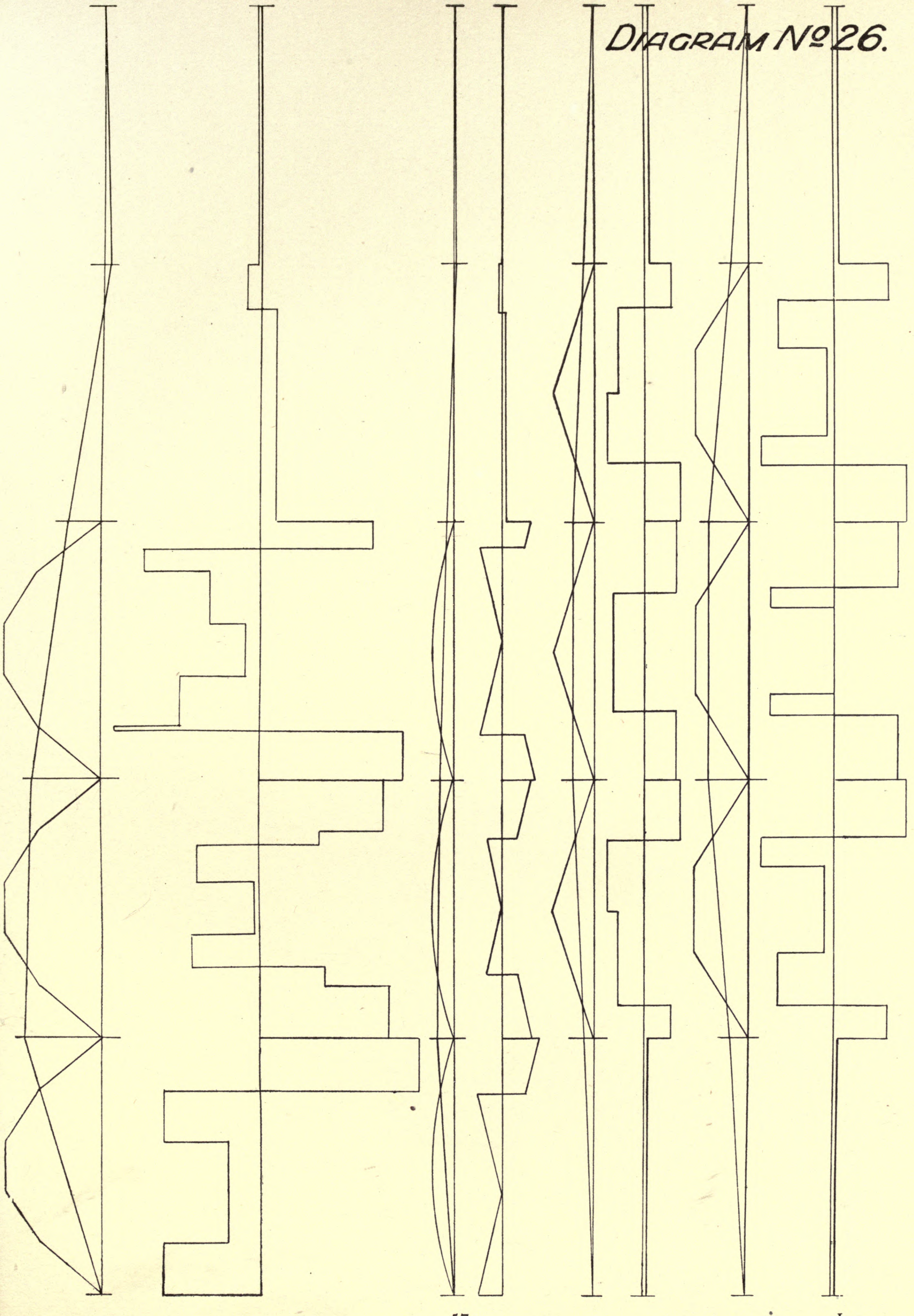
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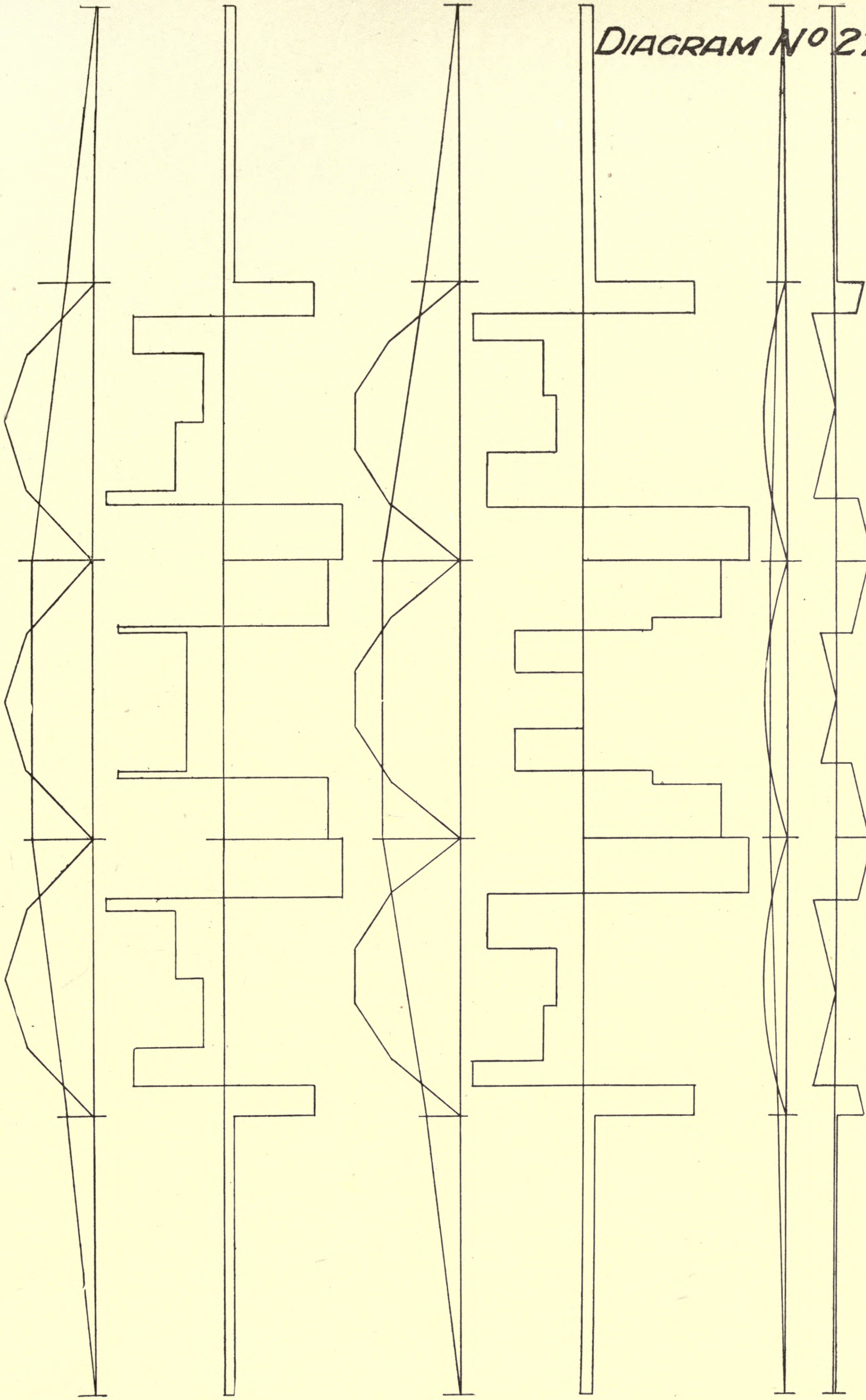

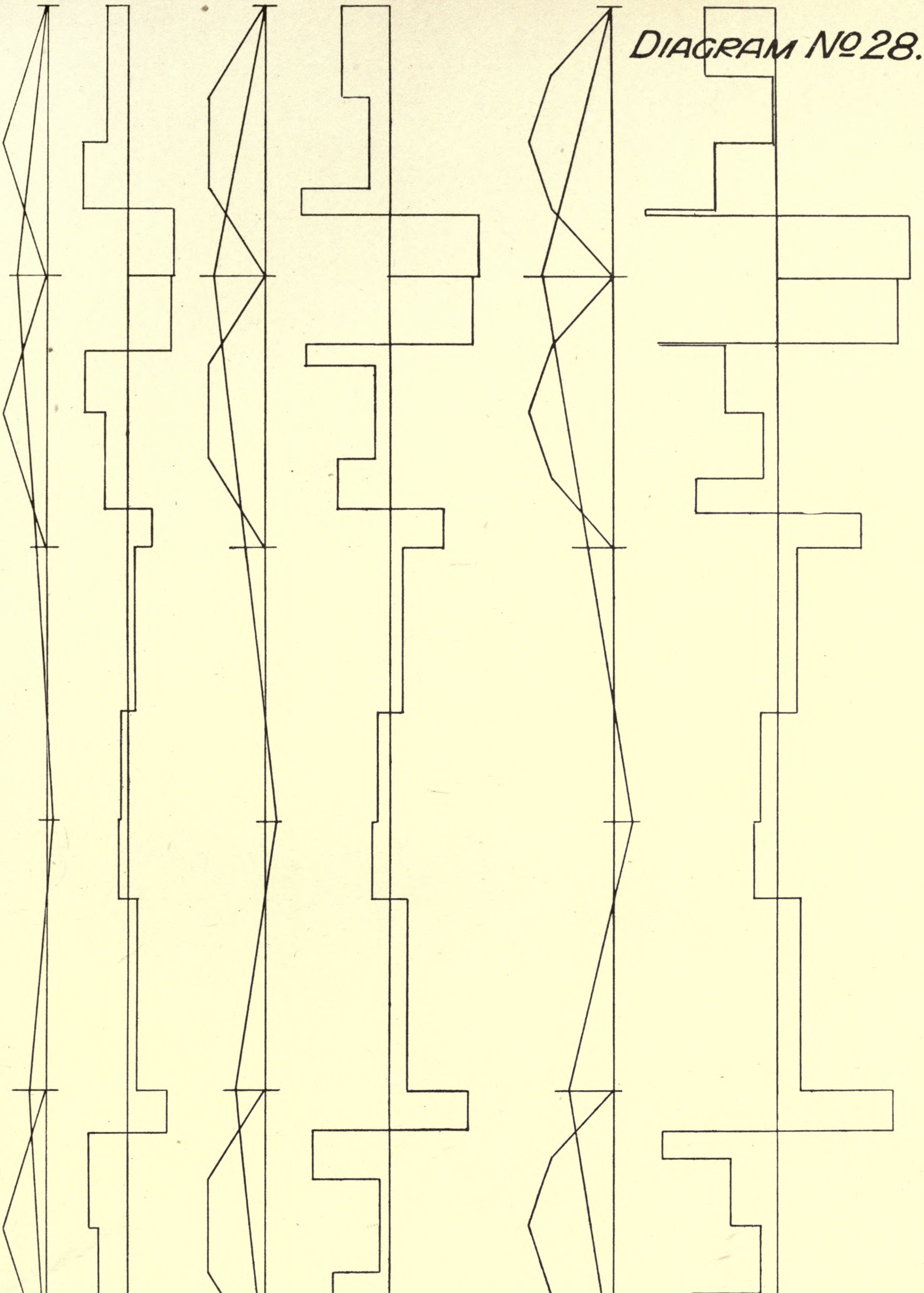
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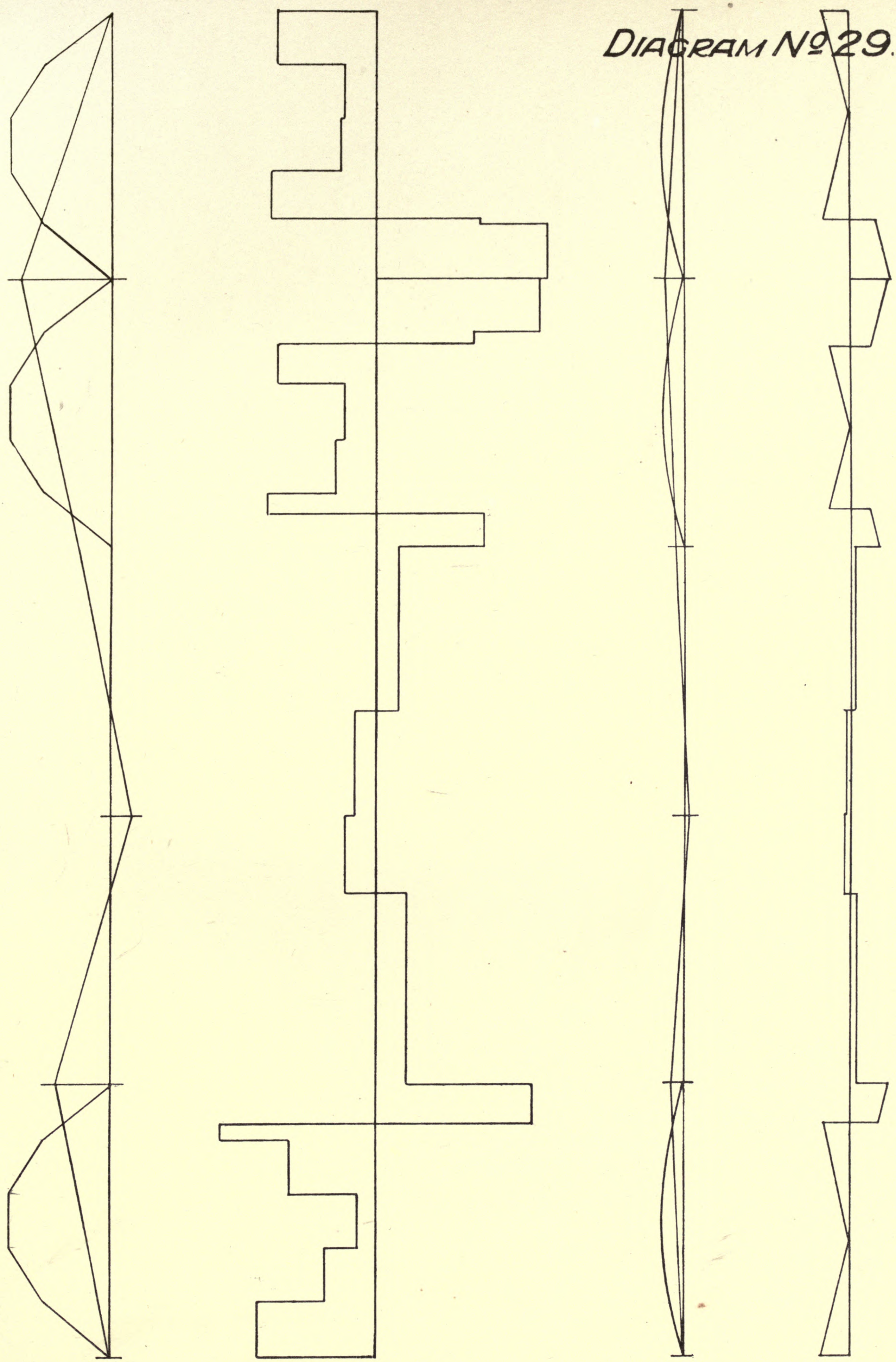
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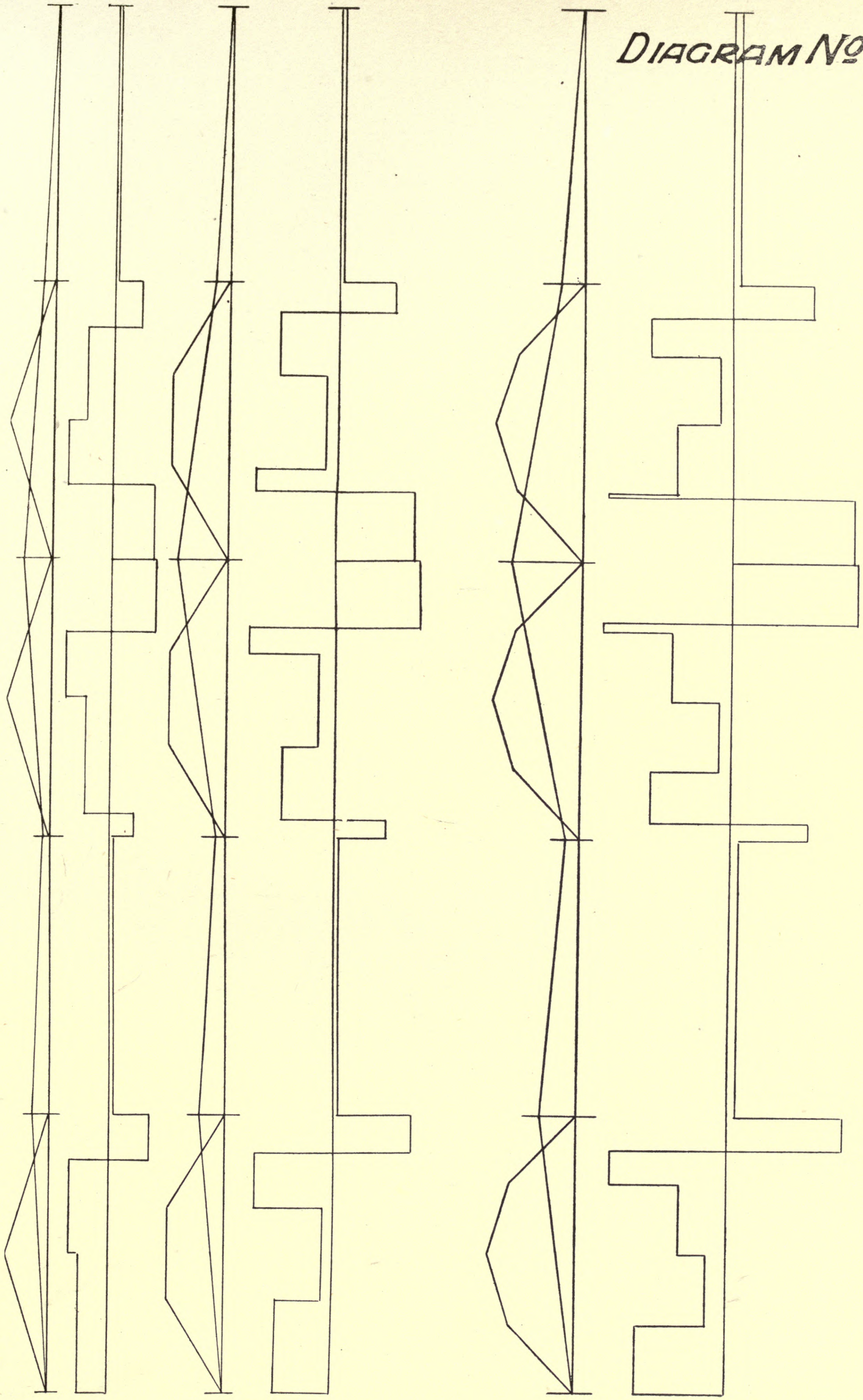

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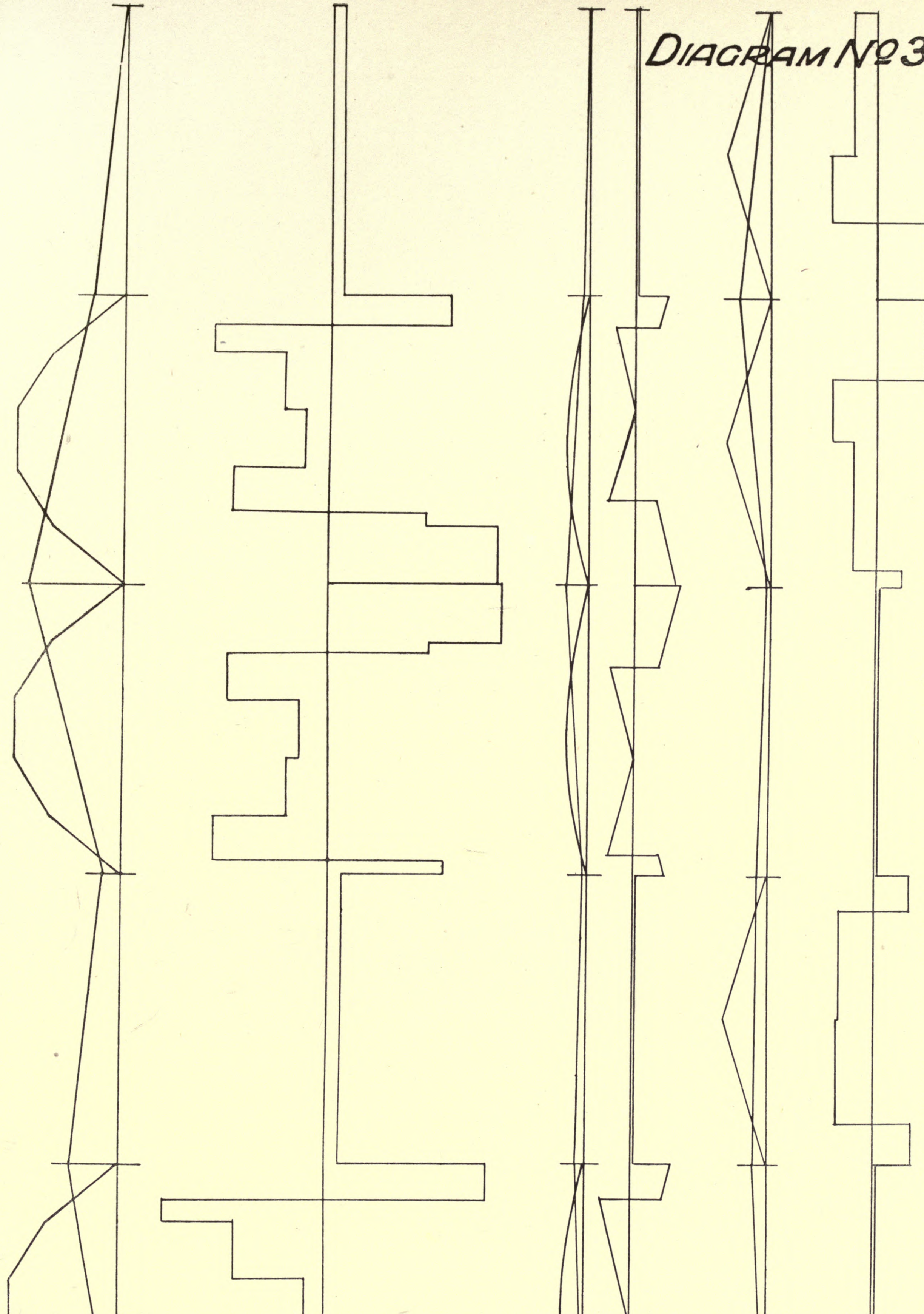

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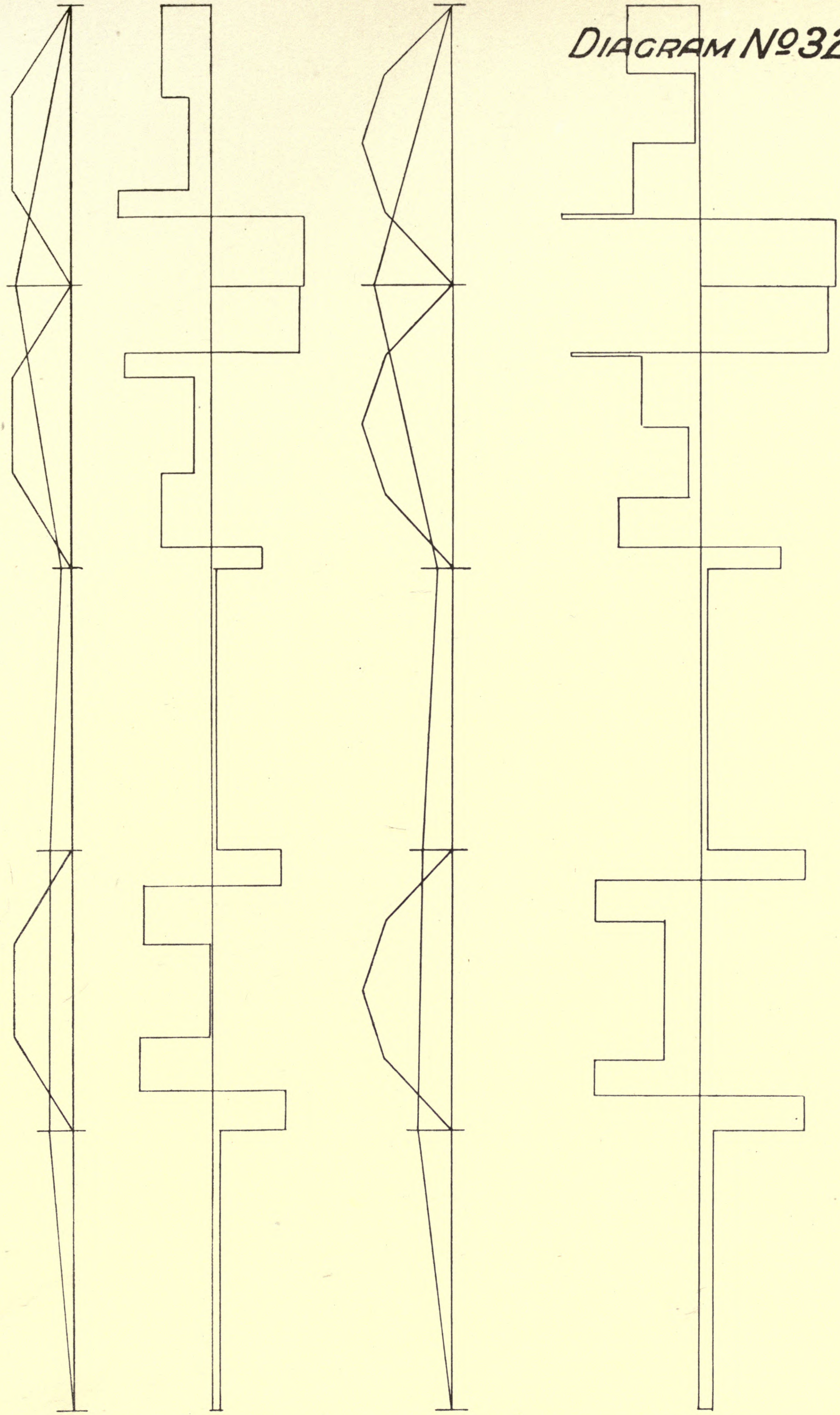

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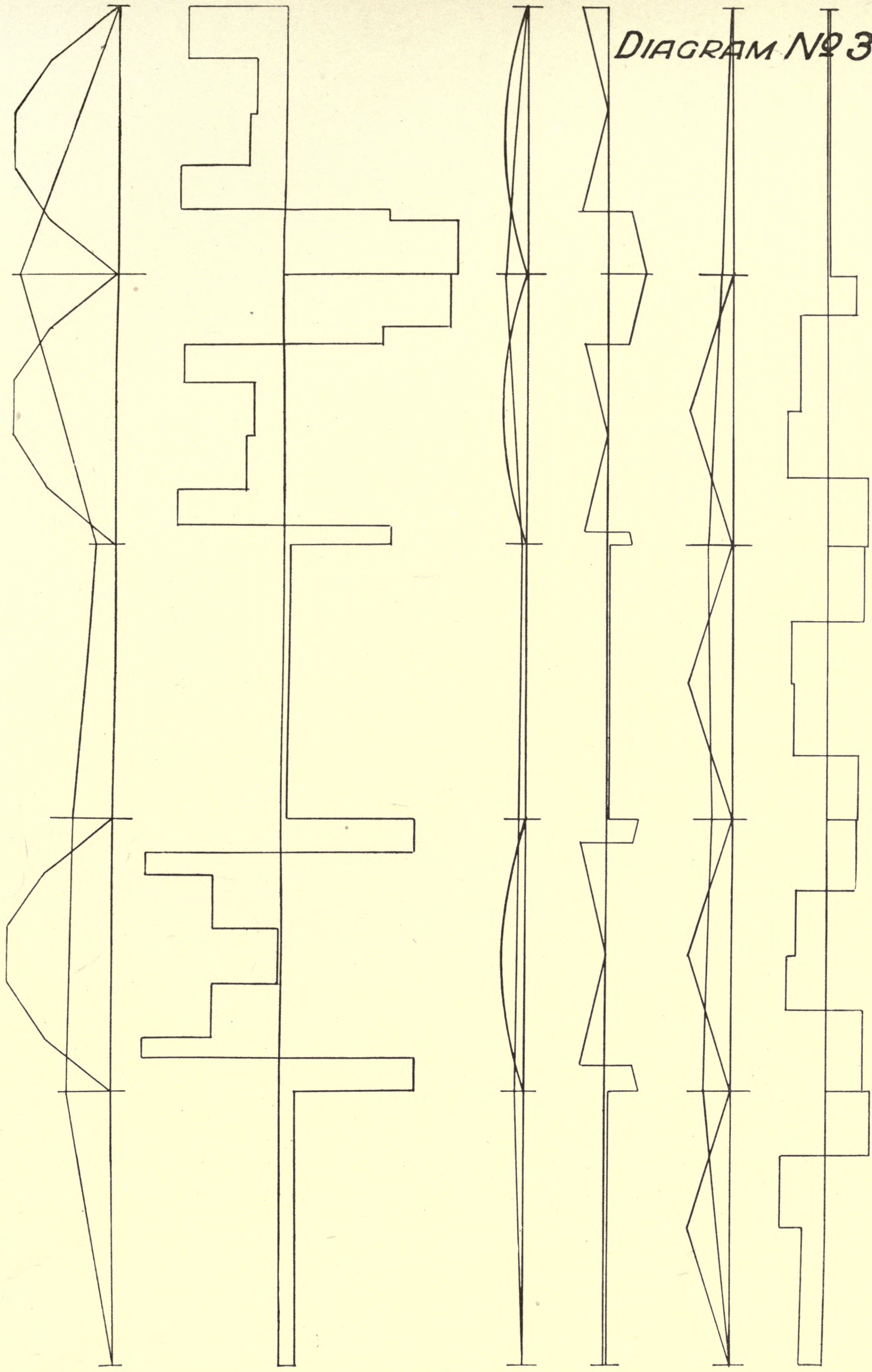

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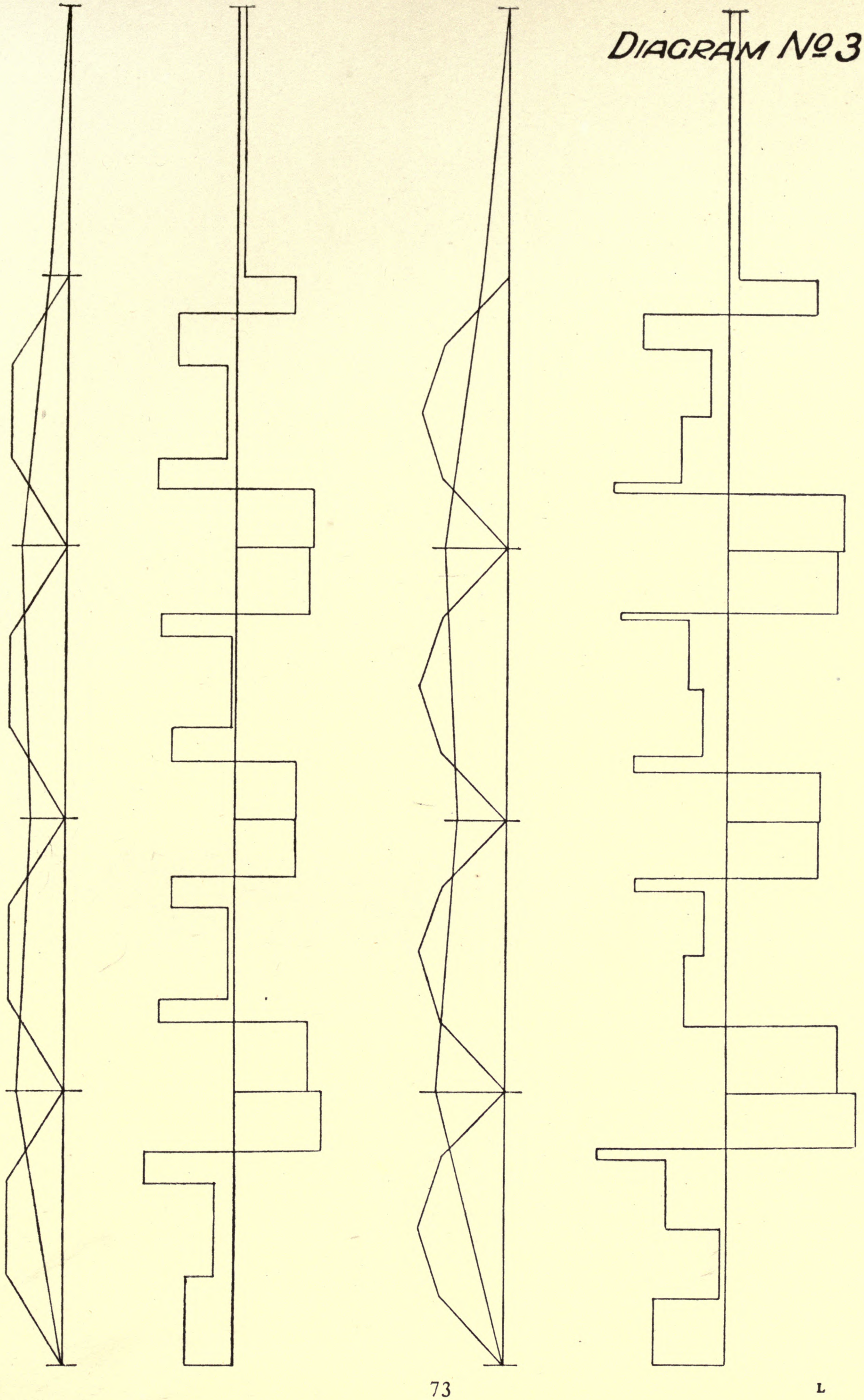

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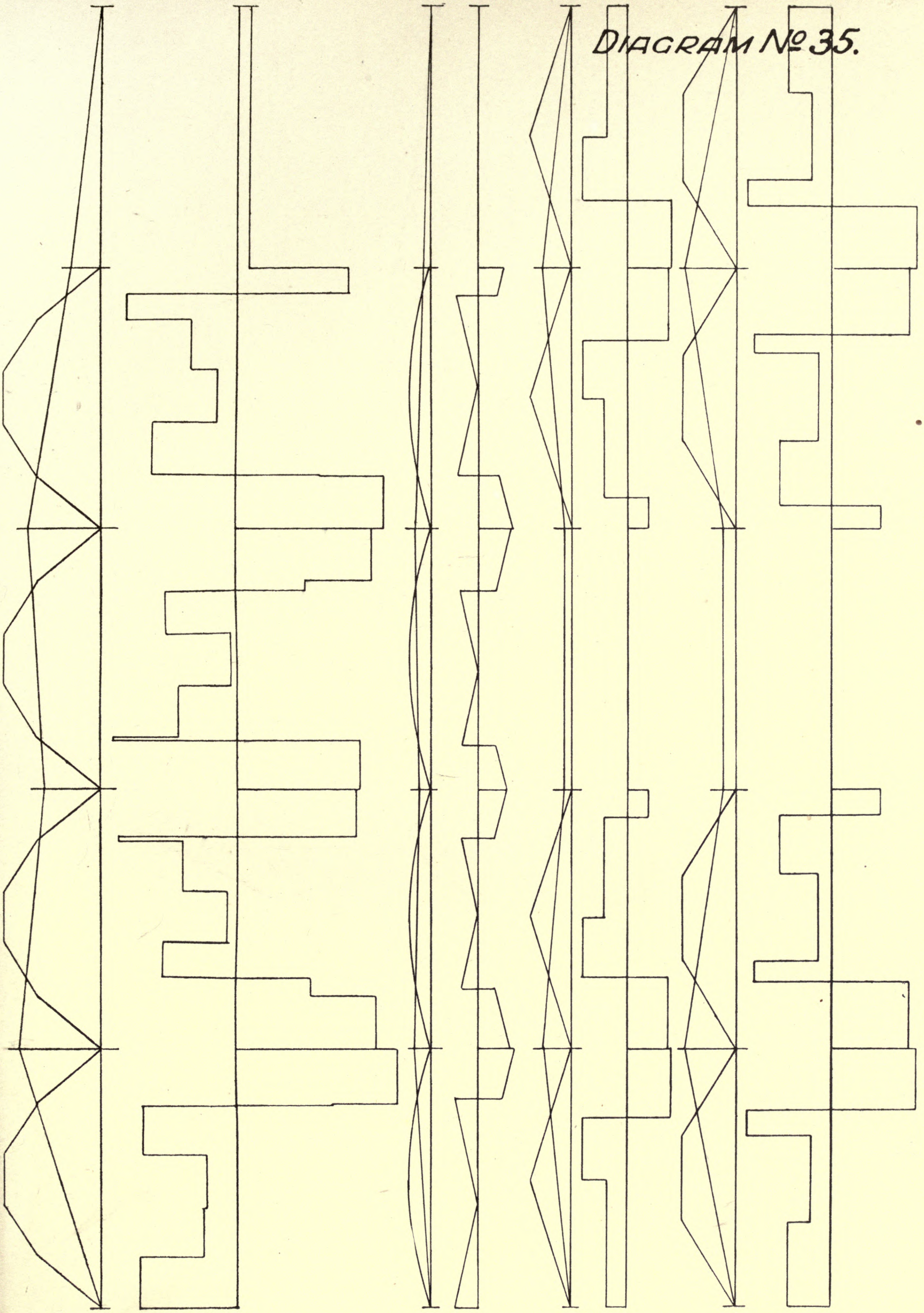
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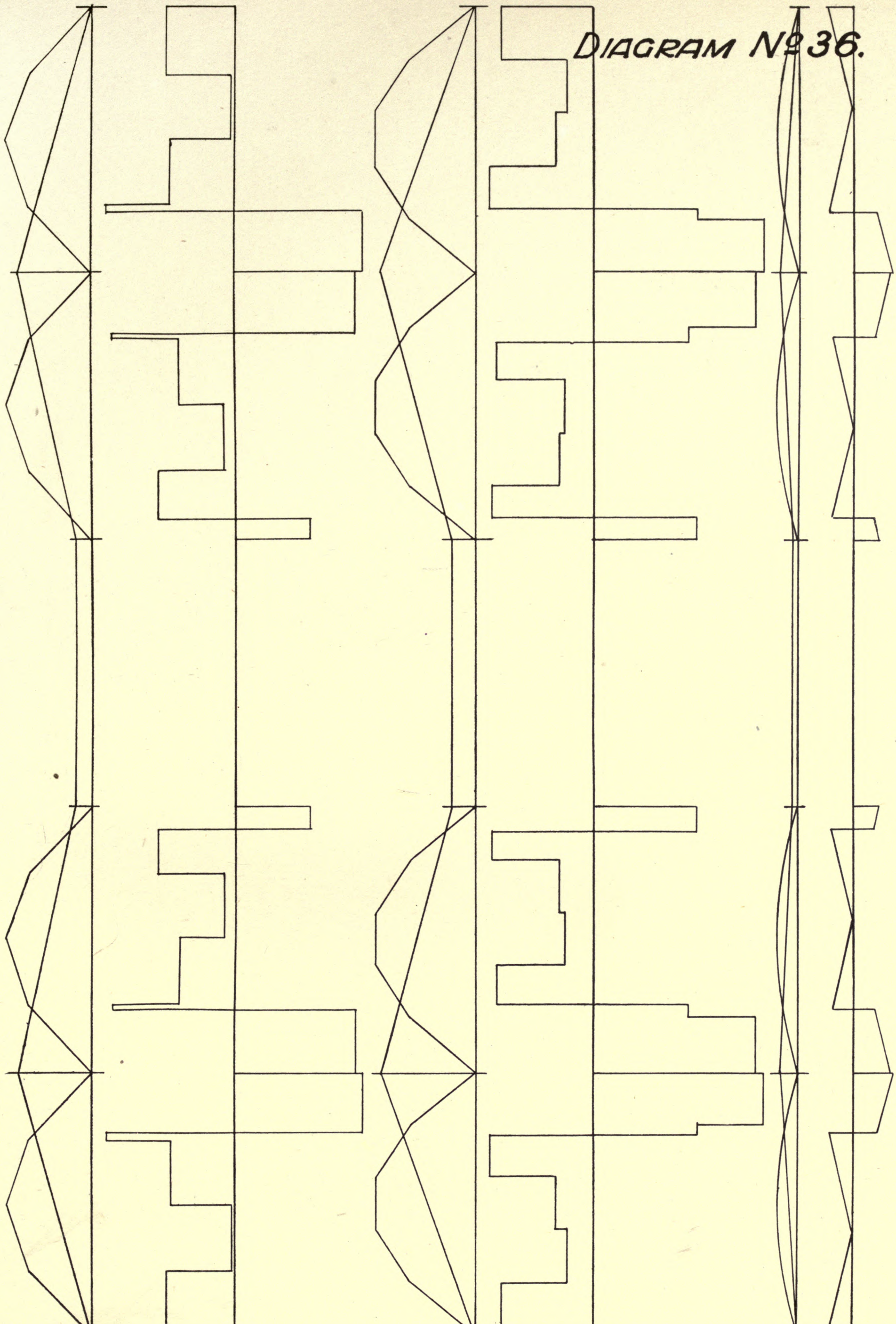
DIAGRAM № 36.



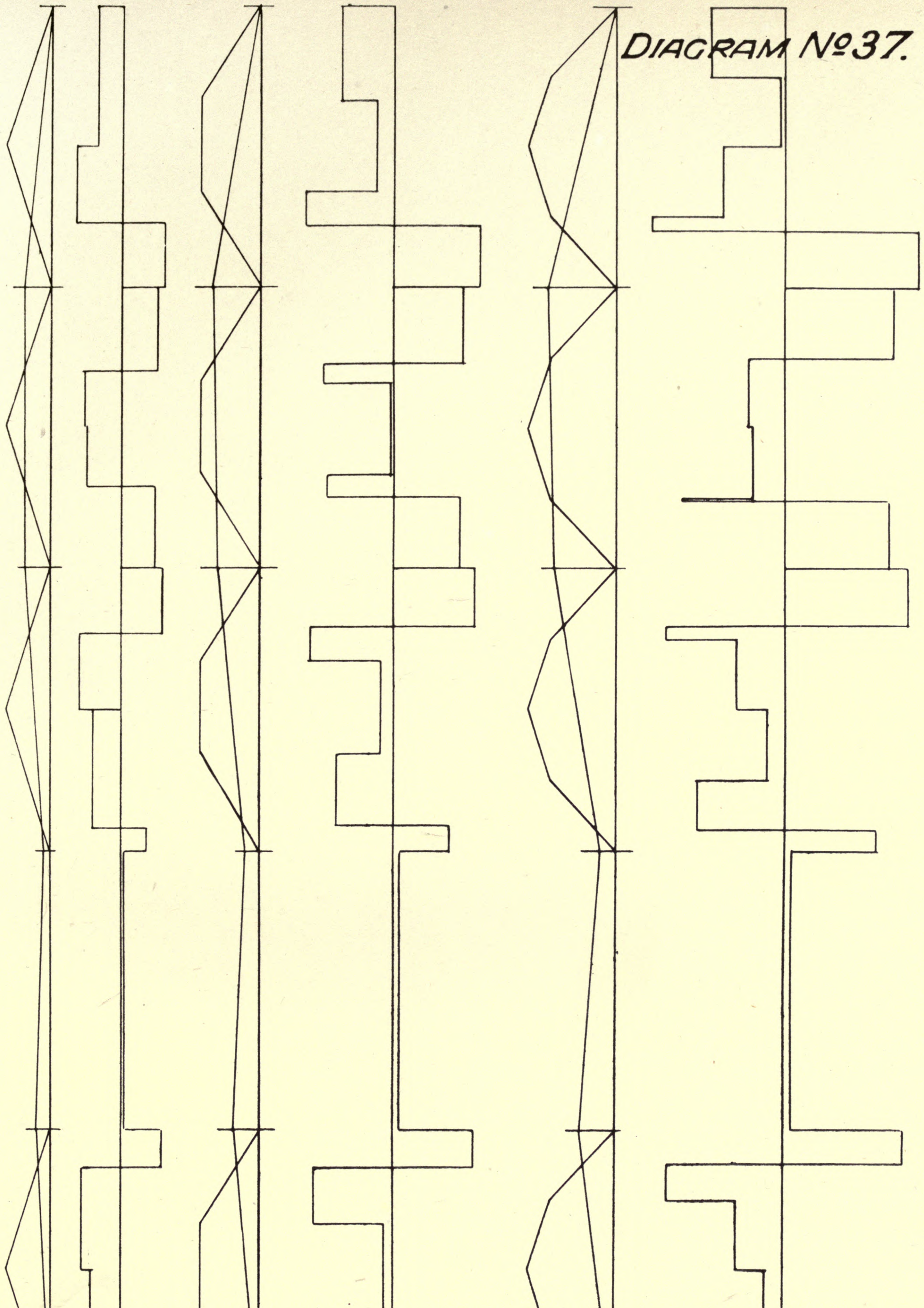


DIAGRAM № 37.

DIAGRAM No. 38.

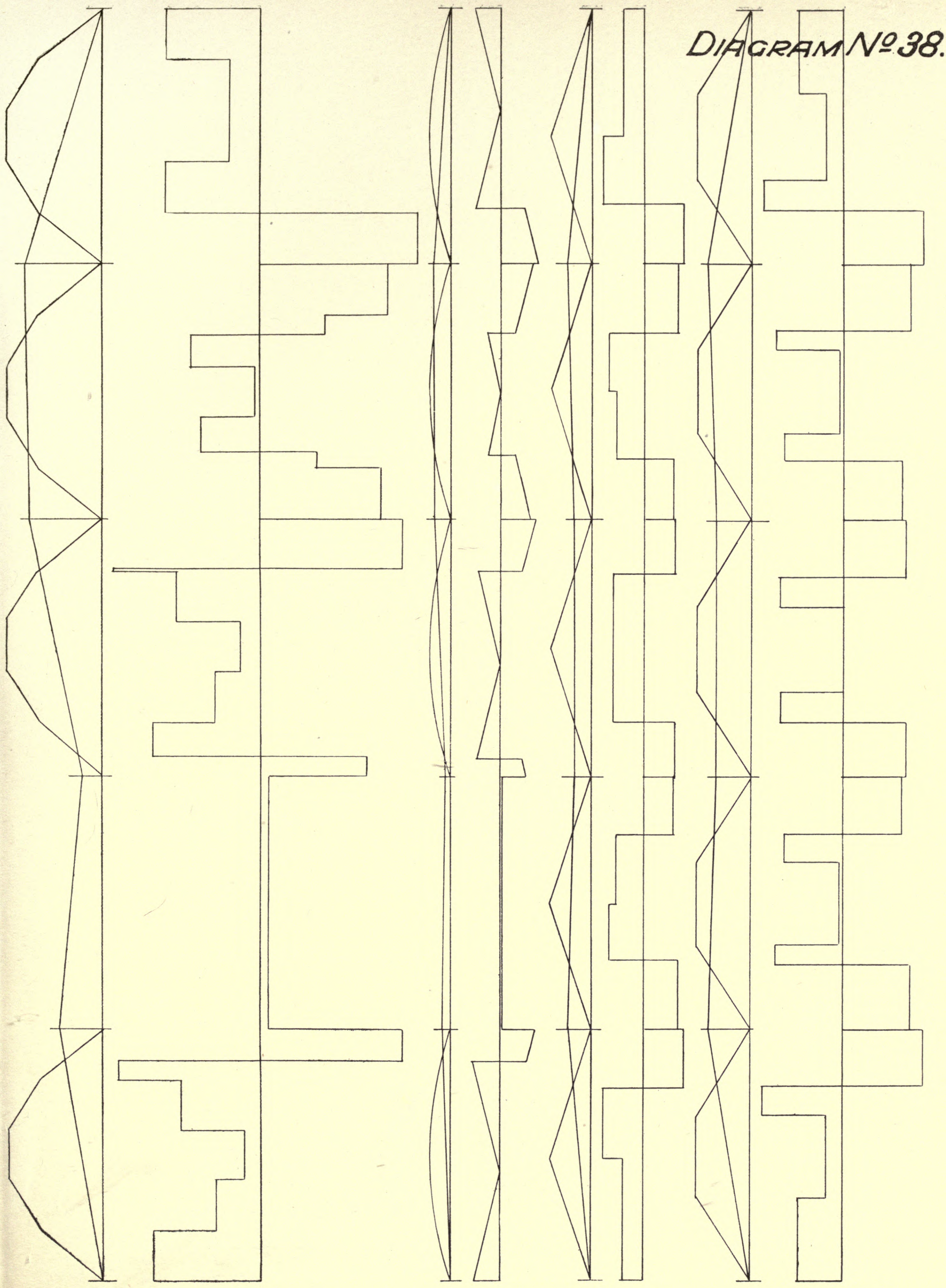
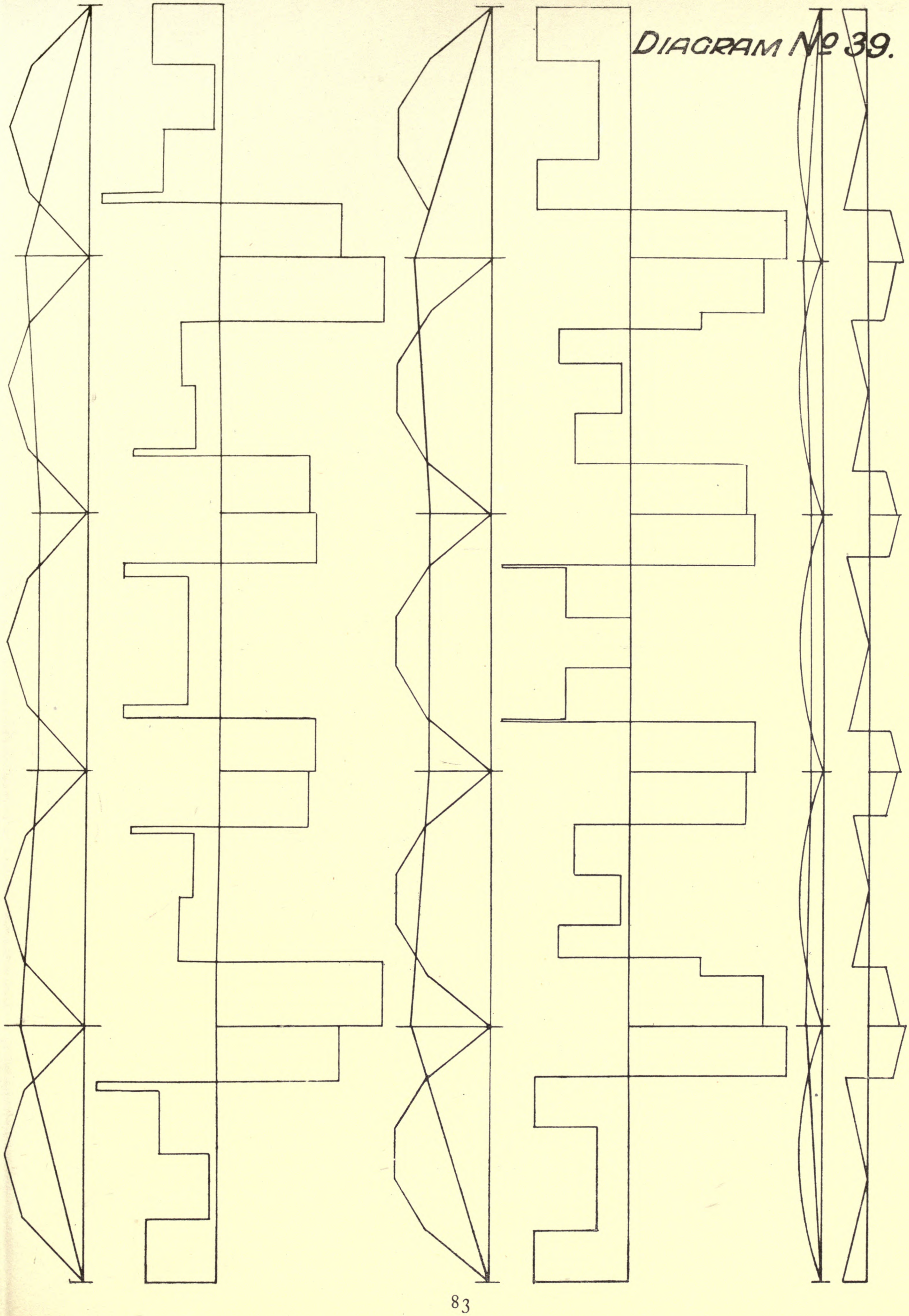


DIAGRAM № 39.



CHAPTER IV.

DIAGRAMS GIVING MAXIMUM LIMITING BENDING MOMENTS AND SHEARS.

FOR the purpose of finally designing any given series of continuous beams, it is necessary to know the maximum positive or negative moments and shears which may be caused by any and every possible arrangement of the assumed load, which is hereafter called the Maximum Limiting Bending Moment or Shear.

The following diagrams, 40–54, have therefore been compiled from the combination of diagrams 1–39.

The primary diagrams 1–39 will be found useful both in designing beams and columns, and also in testing completed work, and show the inefficiency of the tests usually applied, to which further reference is made in Chapter XII.

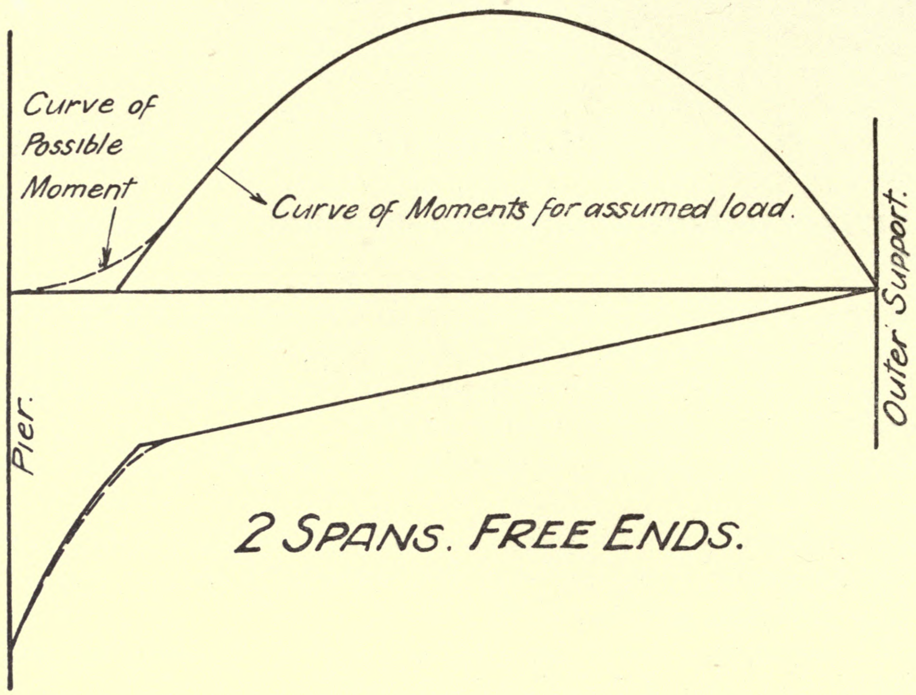
The points of contraflexure having been determined after calculating the pier moments, the shears are derived from the new virtual spans and loading.

It will be noticed that the simple free bending moments in the diagrams 1–69 are erected upon horizontal base lines; and that the base lines of the bending moments derived from them are sloping lines. It has been considered inadvisable to transfer them to a horizontal line again, as the method adopted shows more clearly their derivation, and also enables the effect of a subsidence of the supports to be more readily comprehended.

In all the diagrams and tables the positive moments and shears are considered as being those shown above the derived base lines; and the negative moments and shears those shewn below. By positive moment is meant a moment causing tension in the bottom of a beam, and positive moments and shears are referred to by a + sign; a minus sign — referring to negative moment or shear.

ERROR INVOLVED IN CONSIDERING LOAD ON COMPLETE SPANS ONLY.

It will be observed that only complete spans are considered as loaded with live load. If any possible arrangement of loads had been taken, it would have been necessary to deal with spans loaded in part only, as well as over their entire length. The result of this would be to increase the number of subsidiary diagrams and calculations to an impossible extent, amounting to many thousands; and it has not been considered worth while, in view of the very small increase of moments so derived. To make this point clear and in order to verify the statement, the cases of two and three spans with free ends, and loaded with distributed load, have been worked out; and Fig. 3 shows the comparison of diagrams of bending moment on the two assumptions, the full lines representing the moments due to any possible arrangement of assumed loading on complete spans; and the dotted lines representing the moments due to every possible arrangement of loading with distributed load.


Curve of
Possible
Moment
Curve of Moments for assumed load.
Pier.
Outer Support.
2 SPANS. FREE ENDS.

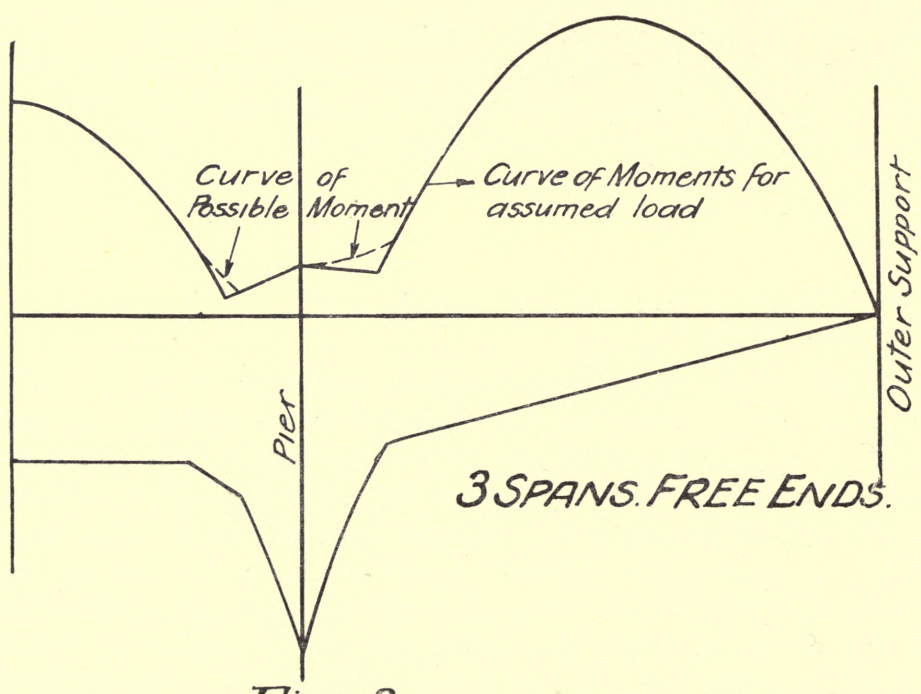

Curve of
Possible Moment
Curve of Moments for
assumed load
Pier
Outer Support
3 SPANS. FREE ENDS.

Fig 3.

It will be seen that the error involved is very trifling, amounting to about 5 % of the simple free bending moment $\frac{w\,L^2}{8}$ as a maximum, and that this occurs where it is to a large extent inoperative due to the contra effect of the dead load.

WORKING STRESSES IN BEAMS.

It will be noted that the beams undergo considerable changes of amount of stress as the arrangement of the loading varies, and under certain conditions very serious changes in the nature of the moments and shears have to be borne, which are not in any way covered by the ordinary assumptions of $\frac{w\,L^2}{12}$ at the supports and at the centres of the spans, unless where there are columns the strength of columns to resist bending is taken into account.

In certain cases, where the live load is small in proportion to the dead load, this point is not of any great importance, but there can be no doubt that in the case of structures designed to carry heavy loads, the disposition of which may vary from day to day, or even from hour to hour, there is considerable risk being run by adopting the ordinary assumptions, and in the author's opinion lower stresses should be adopted.

The following diagrams apply very generally, but it is assumed that the intermediate columns are acting as props only and as rendering no support to the beams in resisting bending, the beams being assumed stiff in relation to the columns. If, on the other hand, we assume that these columns are able to bear considerable bending moment, we could then considerably decrease the range and amount of variation of stress in the beams. The author is of opinion, however, that this is inadvisable ; and advocates the adoption of full reversal of stress in beams due to live load and dead load combined, and that the intermediate columns be considered as carrying no bending moment as far as the beams are concerned. The outside columns, however, must have proper provision made for bending moment as well as direct compression ; and provision must be made in the beams for carrying the reversal of stress in the form of reduced working stresses in the materials.*

The author is of opinion that in buildings of the warehouse class, where the live loads are great in proportion to the dead load, and where great variation in loading is likely to occur or may occur, that full reversal of stress due to live load should be provided for, and that the working stress in the materials should not exceed : for unequal loading—

12,000 to 13,000 lbs. per square inch in steel
450 ,, ,, ,, concrete†

and where the dead load is not less than the live load, or where reversal of stress does not

* It is not intended in this book to deal with the question of the proportioning of resisting moments of sections, and the reader is referred to the many excellent text-books on the subject, amongst which may be mentioned *Reinforced Concrete Design*, by Faber and Bowie, in which a full treatment is given of the method of treating members subject to combined tension and bending, or compression and bending, which occurs in columns.

† The reader is referred to Claxton-Fidler's *Bridge Construction* and to Unwin's *Machine Design*, Part I., for discussion of the question of alternation of stress and Woehler's and Fairburn's experiments.

occur, stresses of 16,000 lbs. in the steel and 600 lbs. in the concrete may be safely adopted for unequal loading.

BENDING MOMENTS IN COLUMNS.

In treating the intermediate columns, if it is desired to give consideration to the bending moment in them (although the reduction in corresponding moments in the beams is not advocated), the column moments may be readily calculated as soon as we know the position of the points of contraflexure in the beams; or in other words the leverage with which the loads on the columns act.

In the case of one span being loaded and the adjacent span being unloaded, the leverage is the distance of the point of contraflexure in the loaded span from the pier.

To the stress in the column from the bending moment must be added the direct compression in the column due to the axial load; and the stress in the column is of course the direct stress in compression due to the axial load + the stress caused by bending, the latter stress being found by the ordinary formula.

$$\text{Stress} = \frac{M y}{I} \quad \text{where } M = \text{moment.}$$

y = distance from N. axis to extreme outer surface of column.

and I = moment of inertia of section of column.

There are in general four cases to give the nature of the moments and shears in the column.

(*a*) One floor. Column fixed at top and free at bottom.
(*b*) „ „ „ „ bottom.
(*c*) Two floors or more. „ „ „ „ both floors loaded unequally in same direction.
(*d*) „ „ „ „ „ „ „ „ both floors loaded unequally in opposite directions.

The bending moments induced in the outer columns depend upon the stiffness and length of column, and stiffness and span of beams, and proportion of total load to dead load, and also upon the method of fixing the ends of the columns.

For ratios of $\frac{\text{Total Load}}{\text{Dead Load}}$ up to 4, the bending moments in the columns (when the beams are carrying equally distributed load) can be sufficiently accurately expressed in terms of W L, as follows:—

Let W = Total load on beam in lbs., dead and live combined.

L = Span of beam in inches.

$$I_c = \frac{\text{Moment of inertia of column in inch units.}}{\text{Length of column in inches.}}$$

$$I_b = \frac{\text{Moment of inertia of beam in inch units.}}{\text{Span of beam in inches.}}$$

Table No. 1.

FOR SINGLE LENGTH OF COLUMN, BOTH ENDS FIXED.

RATIO $\frac{I_c}{I_b} \times 4$	B.M. IN OUTER COLUMN.
0·25	0·0058 W L
0·5	0·0117 W L
0·75	0·0175 W L
1·0	0·0233 W L
1·25	0·0292 W L
1·5	0·035 W L

For intermediate length of a flight of columns × B.M. in Table by 1·5.
For single length of column, bottom end free × B.M. in Table by 0·75.

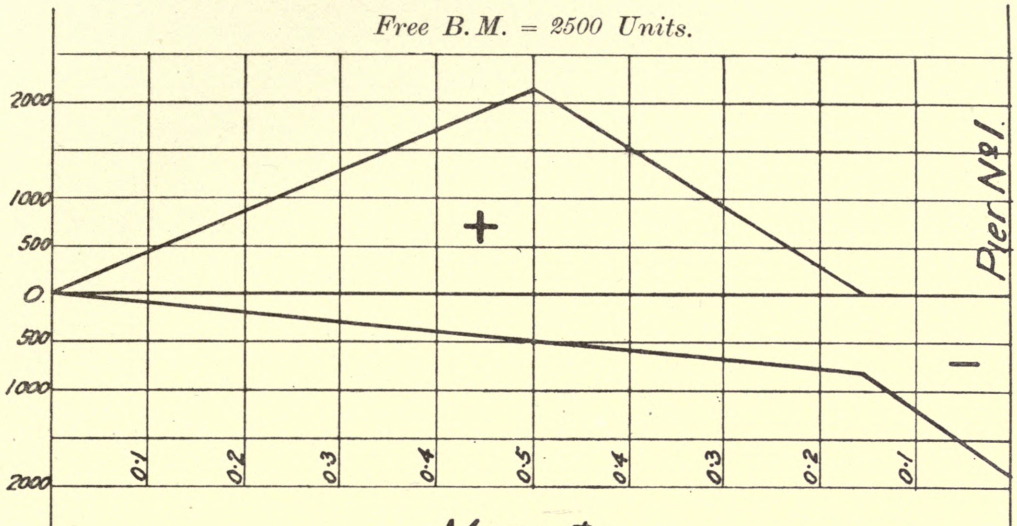
Any intermediate values may be taken between the figures given in the Table.

If it is required to find the eccentricity of load in inches this may be obtained with sufficient accuracy by dividing the B.M. in the column by 45% of the total load on the outer span of the beam; that is to say, by assuming the outer end of the beam to be freely supported. The error in the foregoing Table is on the safe side for small ratios of $\frac{\text{Total Load}}{\text{Dead Load}}$; and is not more than about 5 per cent. deficient when the ratio is 4 for limits of $\frac{4I_c}{I_b}$ up to 1; beyond which value it is again on the safe side.

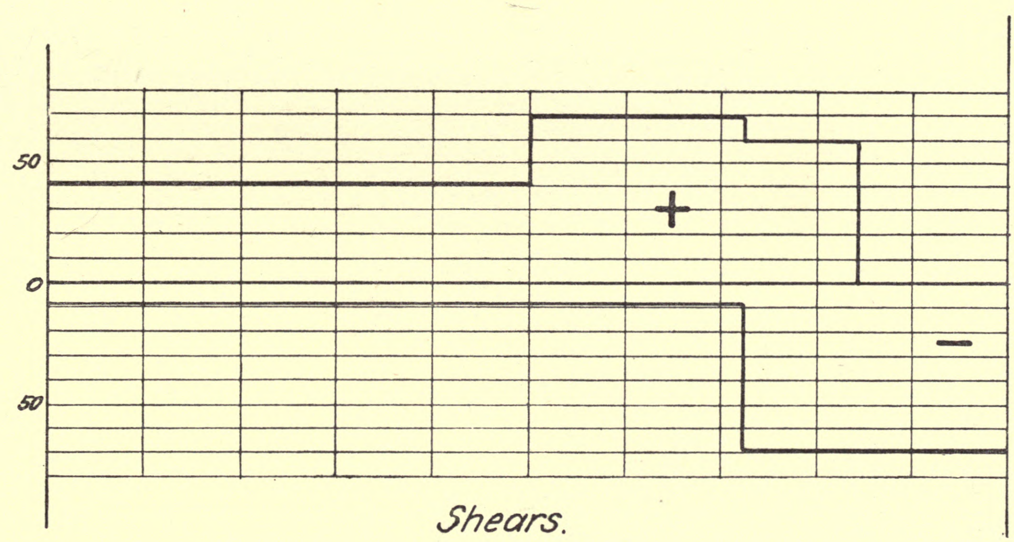
Attention is drawn to the fact that if reduction in the theoretical moments in the beams is made on account of the ability of the column to resist bending, it is not then sufficient to consider the columns as carrying direct load only; but they must be proportioned to carry bending moment as well, and the stress in the column in the case of unequal load may occasionally exceed the stress due to direct load even with the building fully loaded.

DIAGRAM № 40.

TWO SPANS.



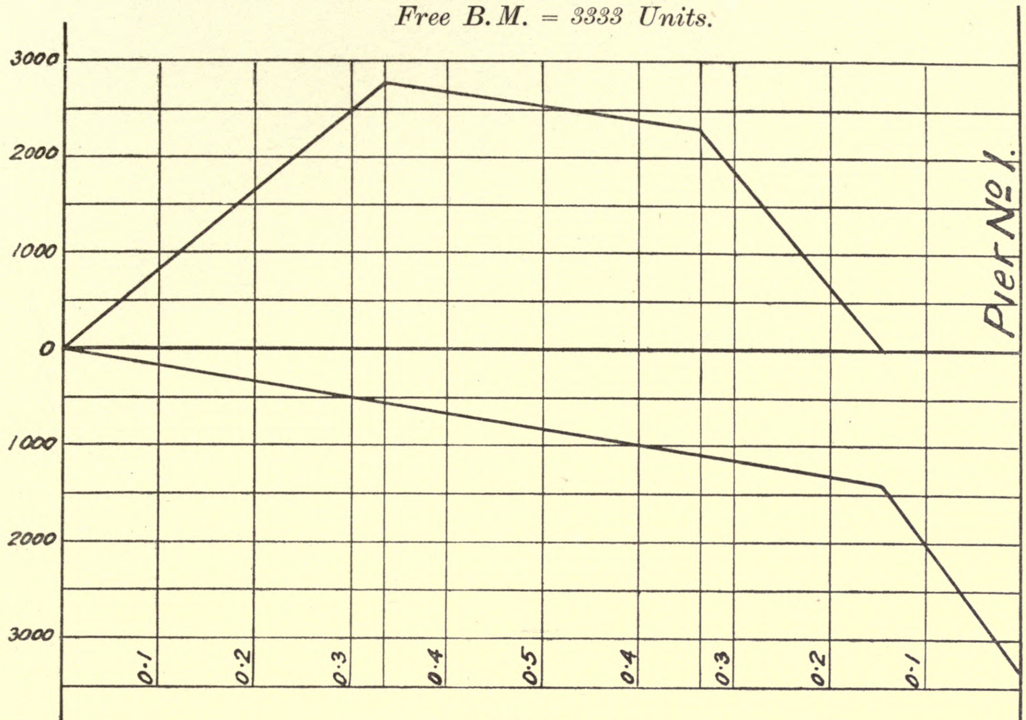
Moments.



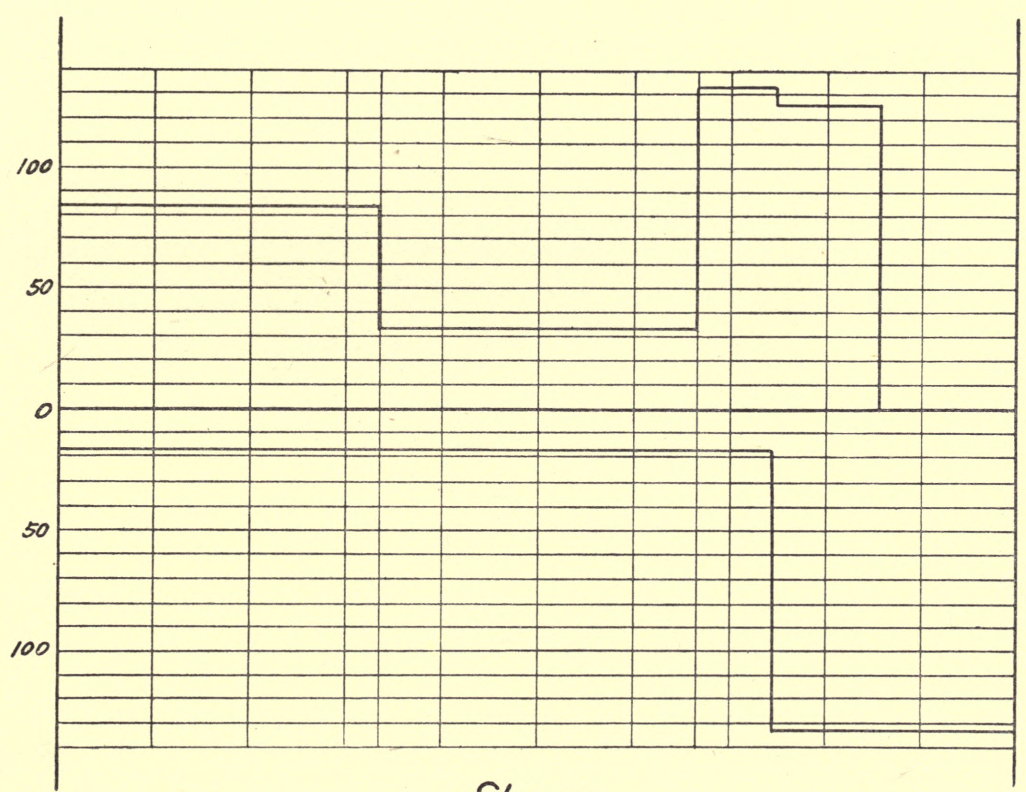
Shears.

Diagram No. 41.

Free B.M. = 3333 Units.

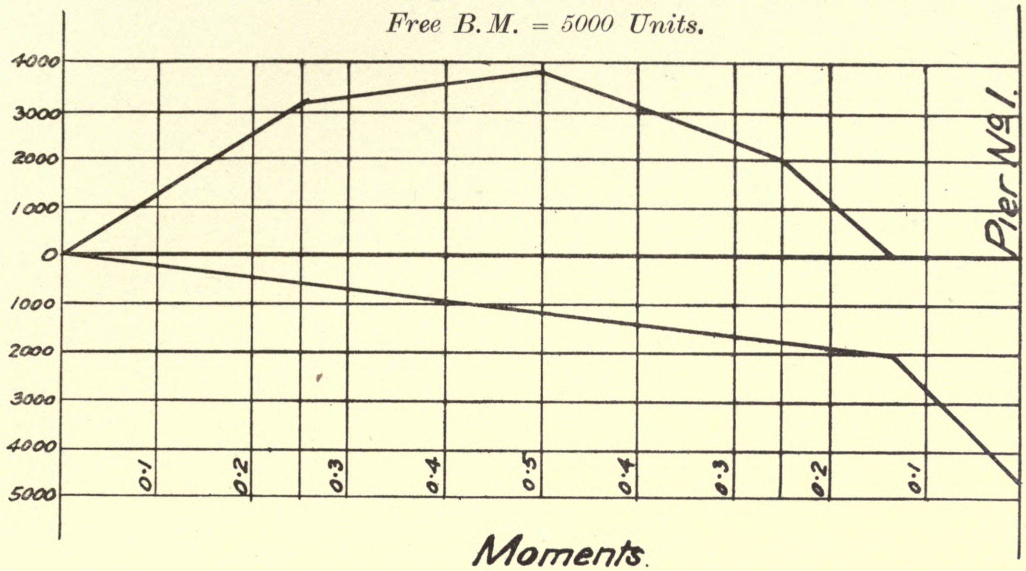


Moments.

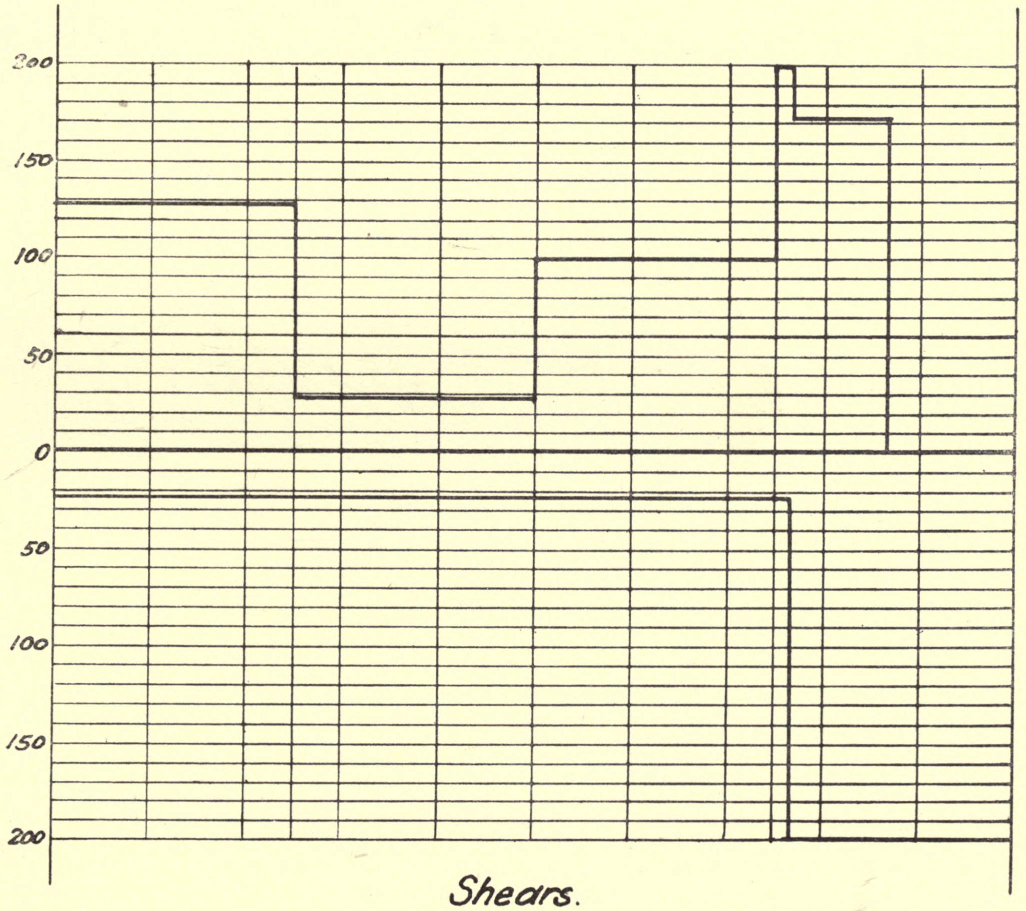


Shears.

DIAGRAM No. 42.

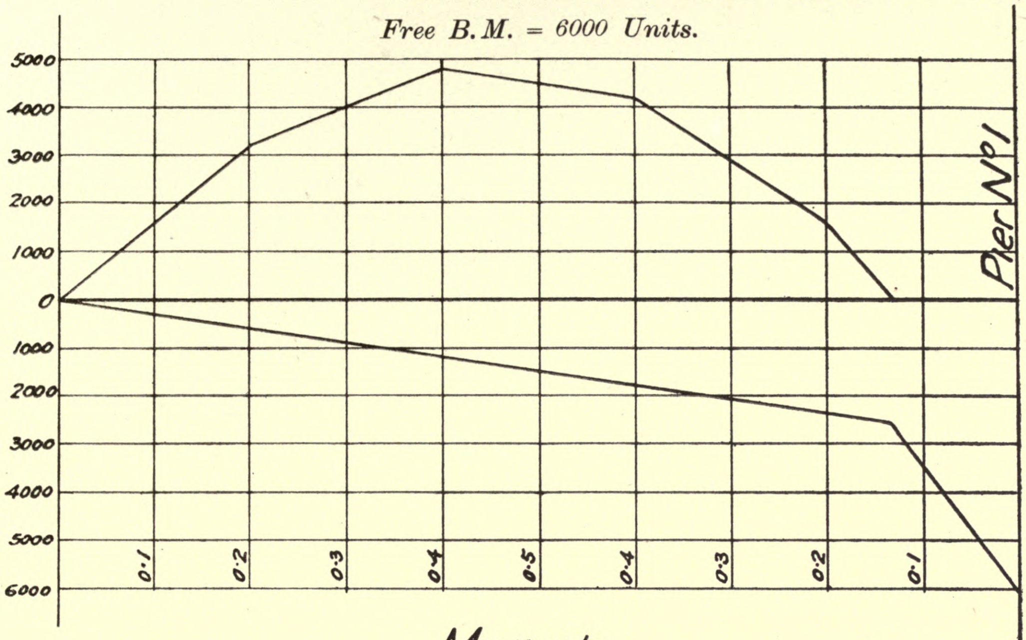


Moments.

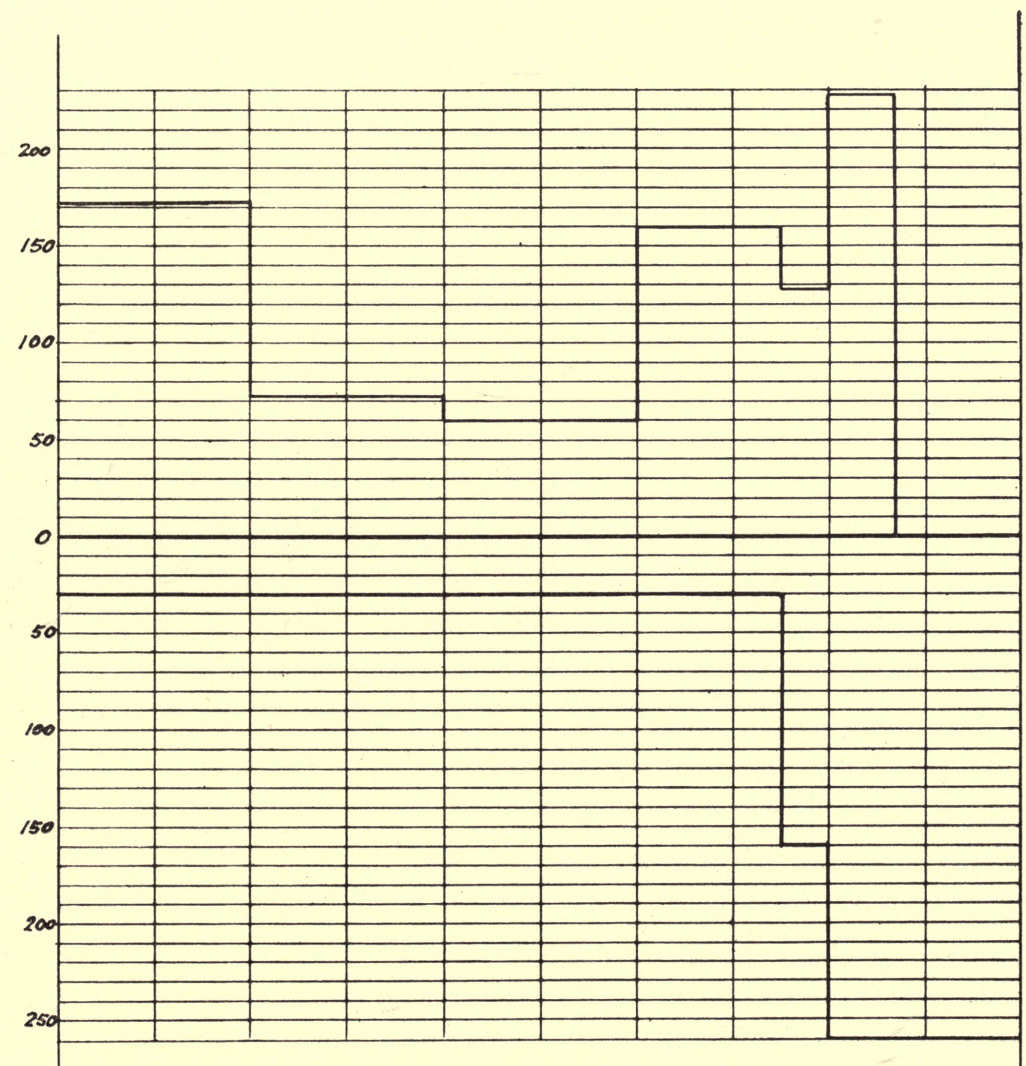


Shears.

DIAGRAM No. 43.

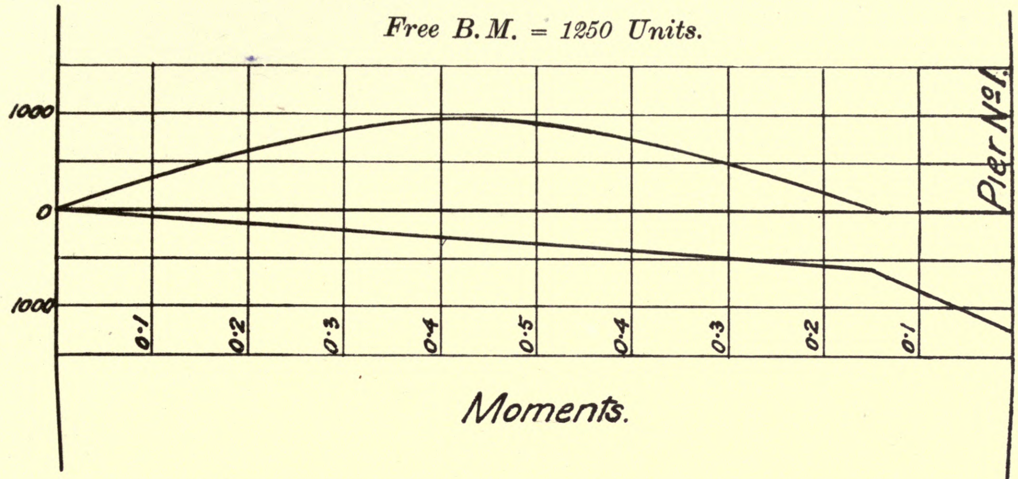


Moments.

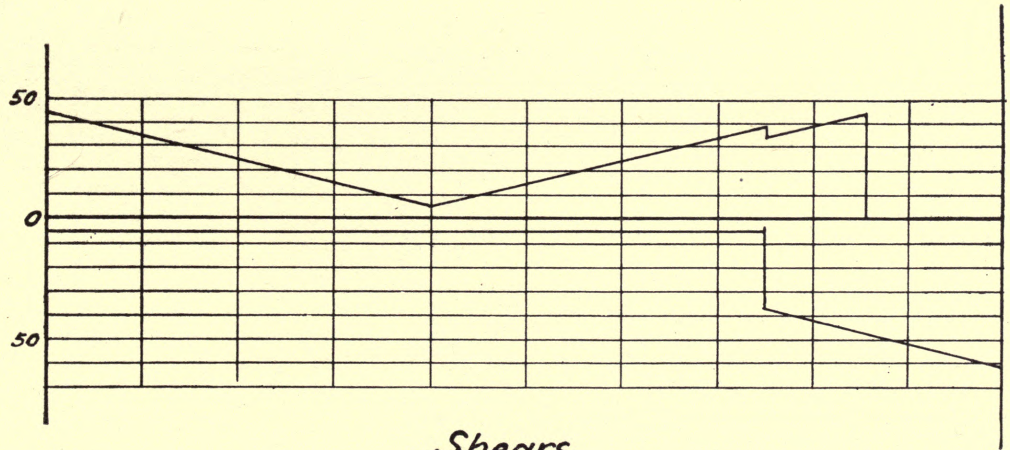


Shears.

Diagram No. 44.



Moments.



Shears.

DIAGRAM No. 45.

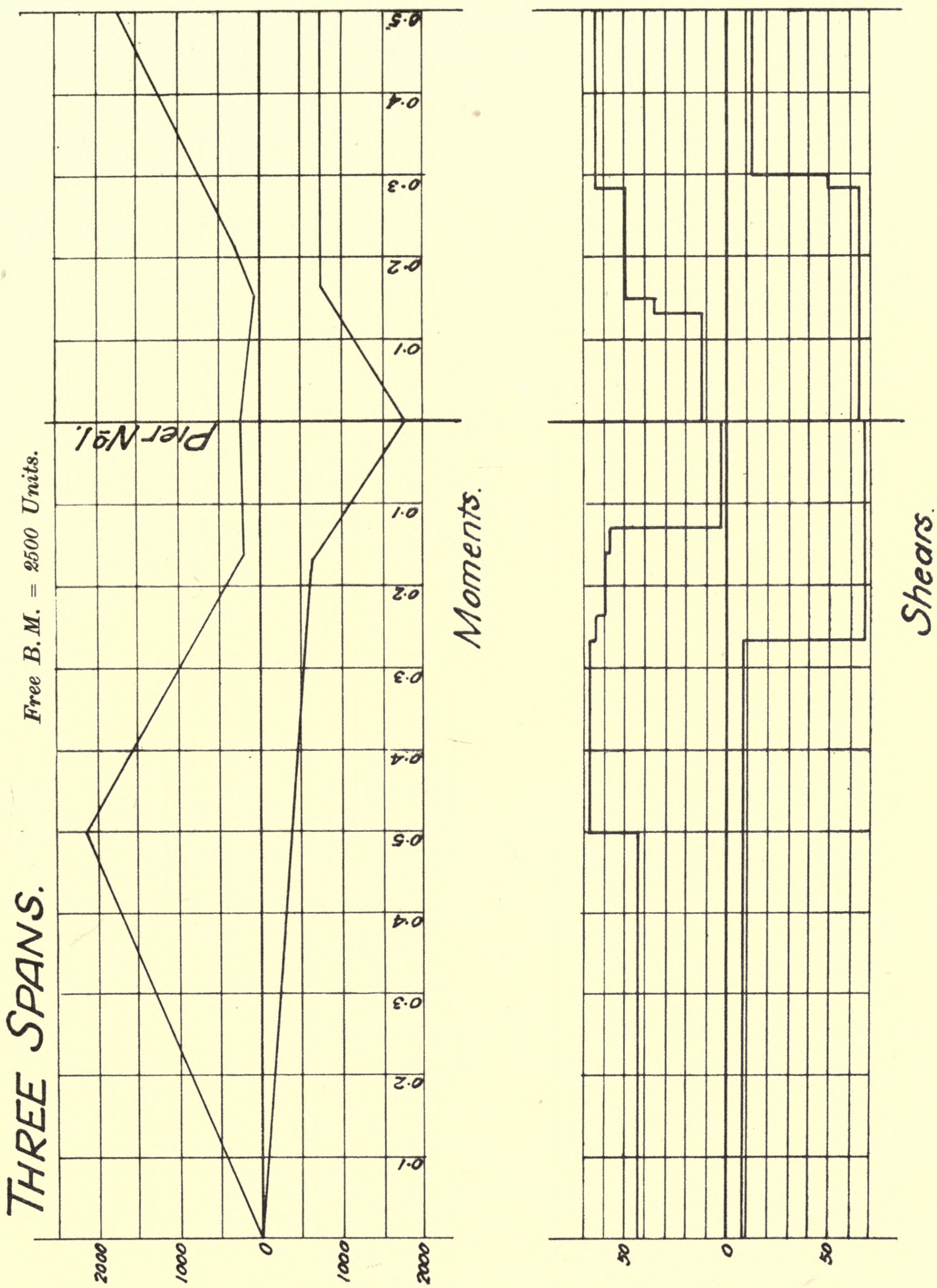
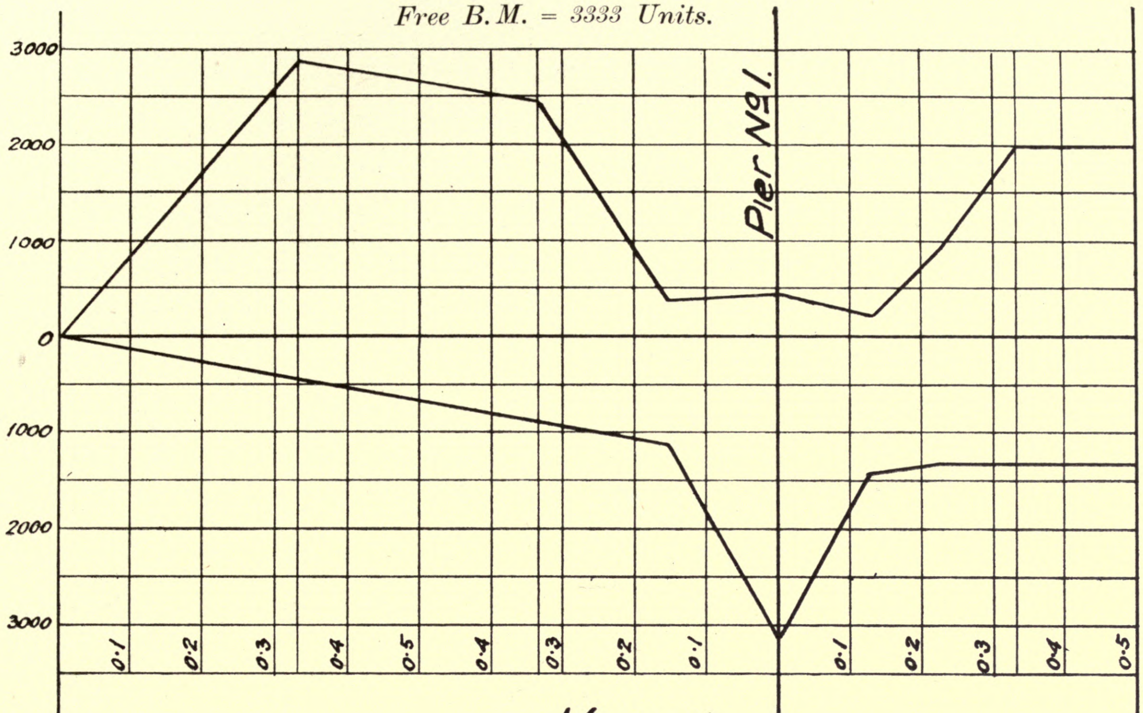
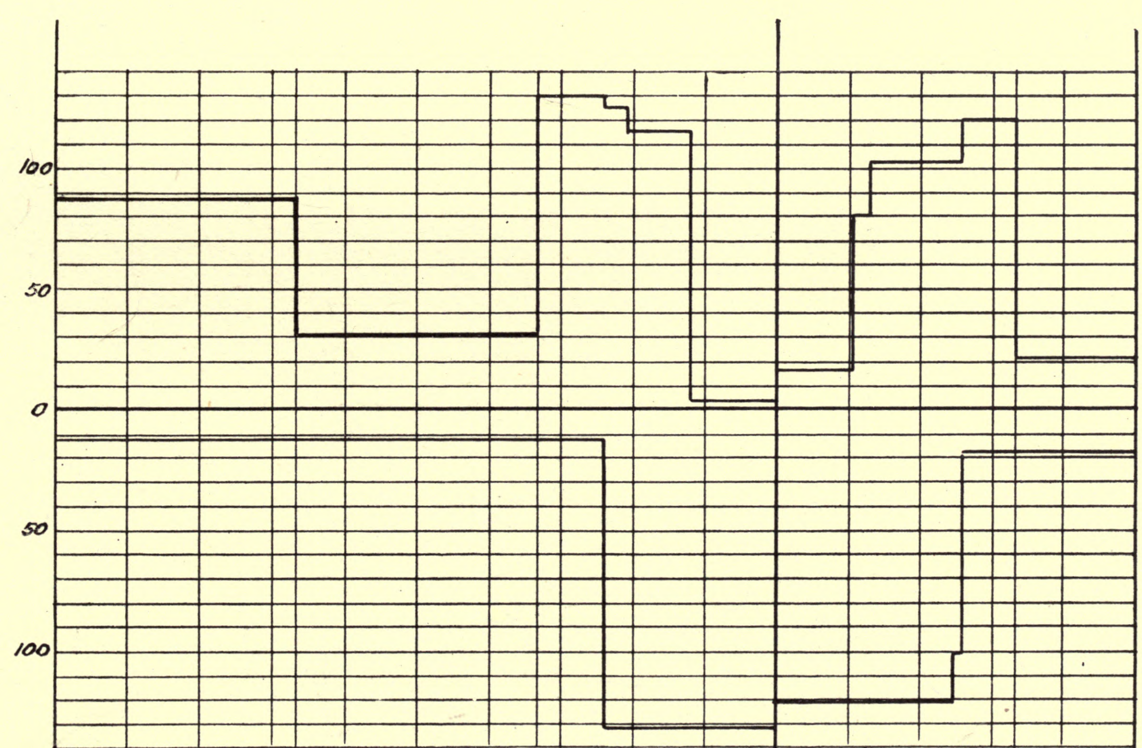


DIAGRAM No. 46.



Moments.



Shears.

DIAGRAM No. 47.

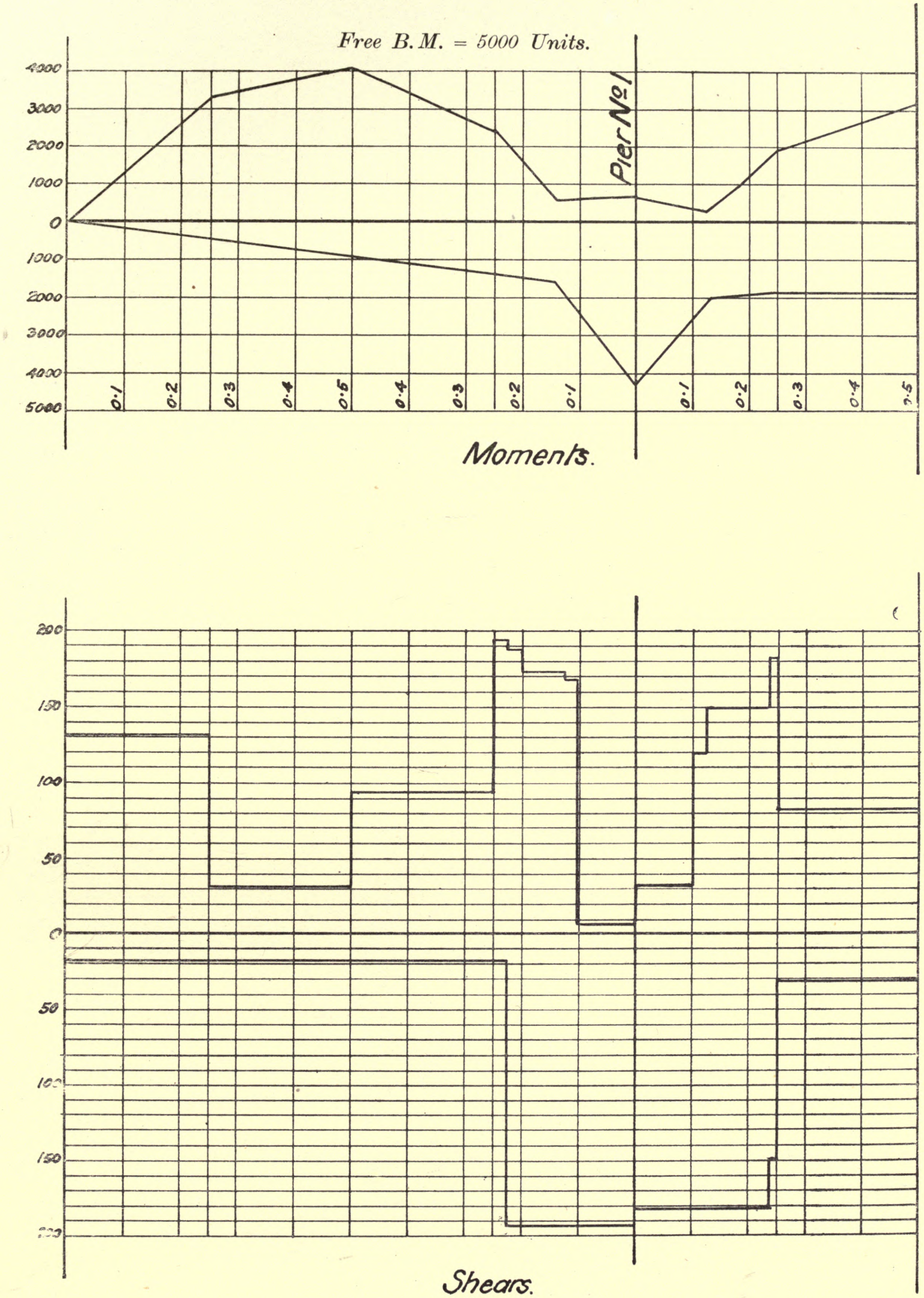


DIAGRAM No 48.

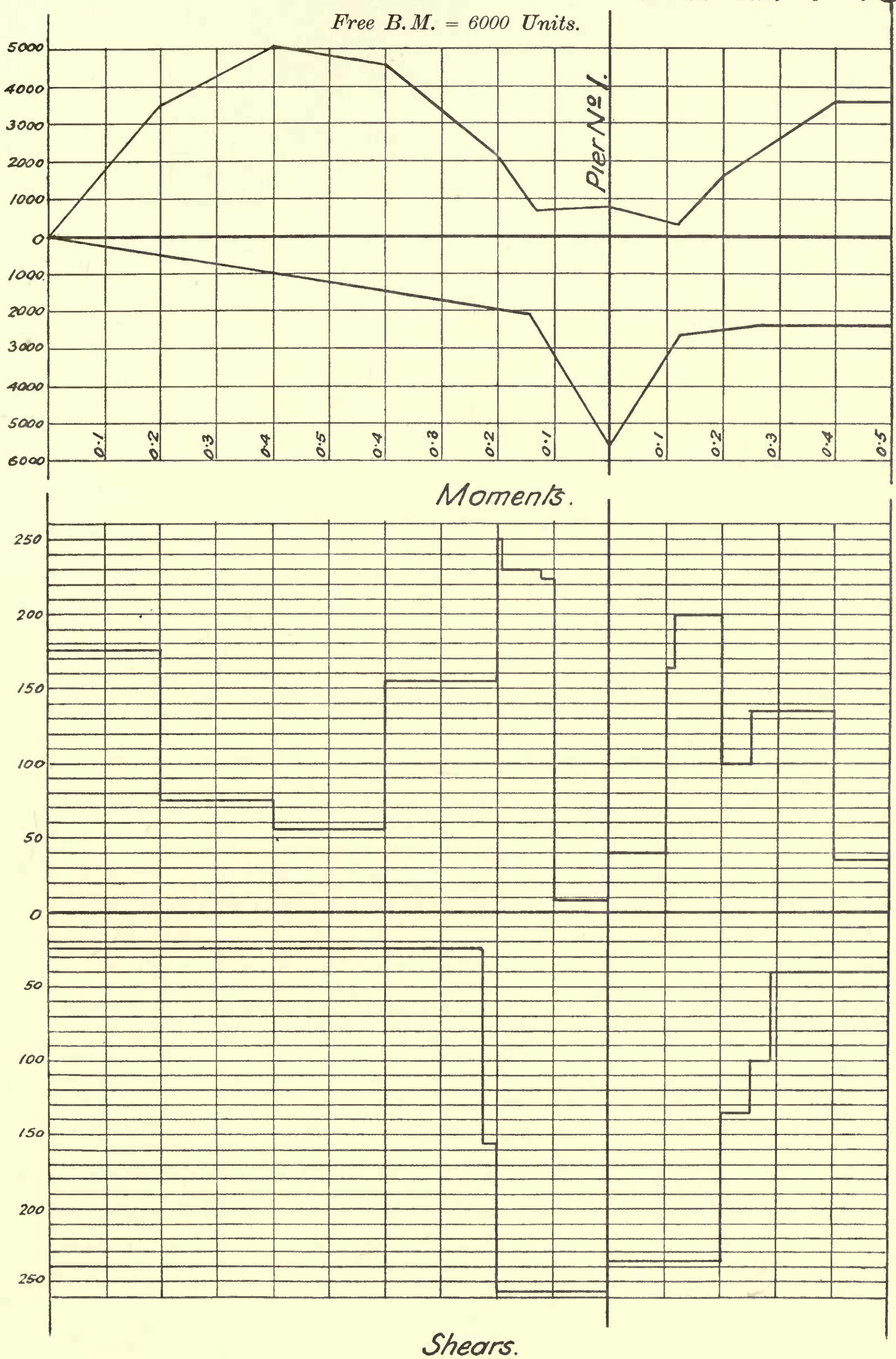


DIAGRAM Nº 49.

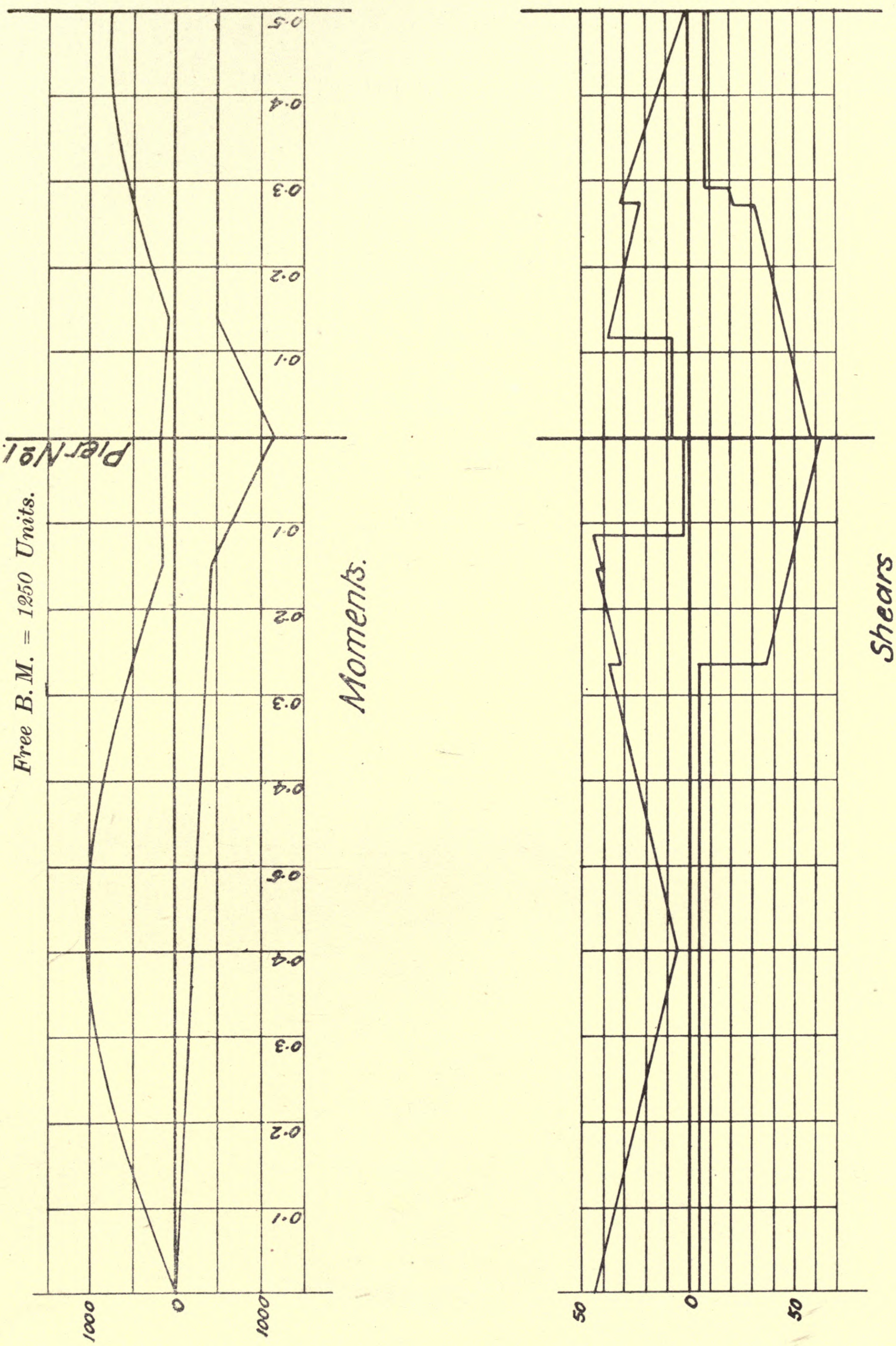
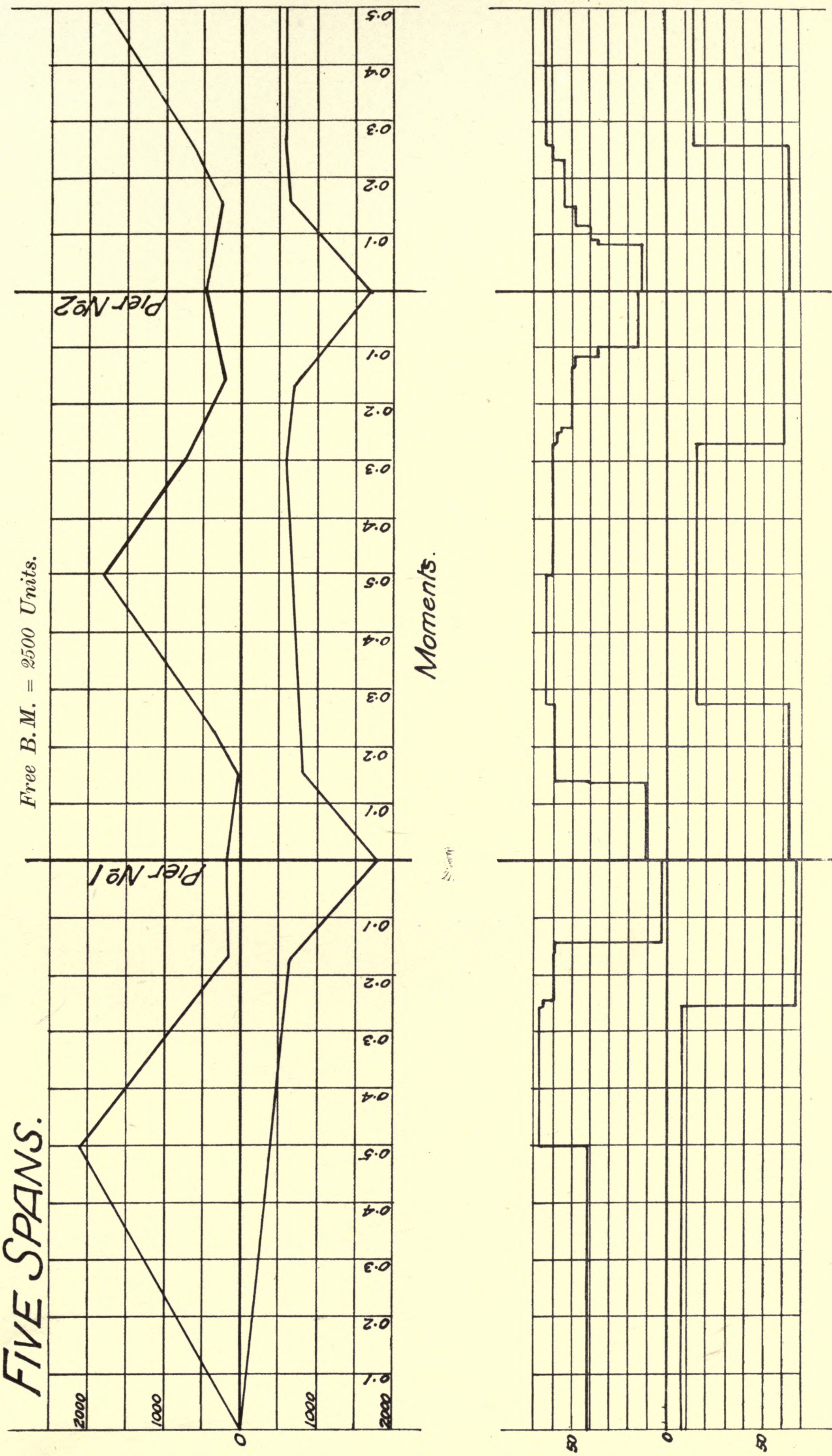
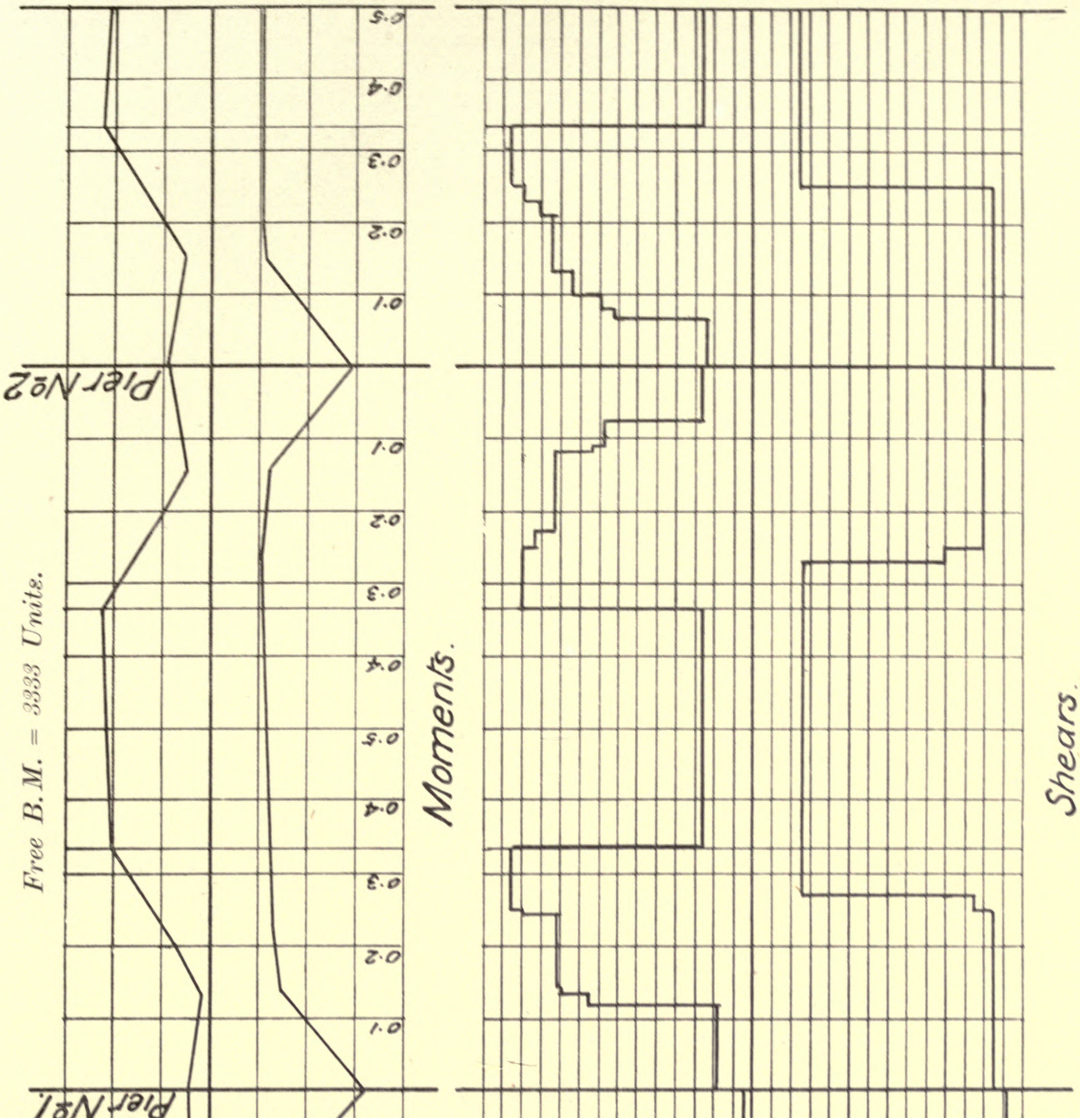


DIAGRAM №50.

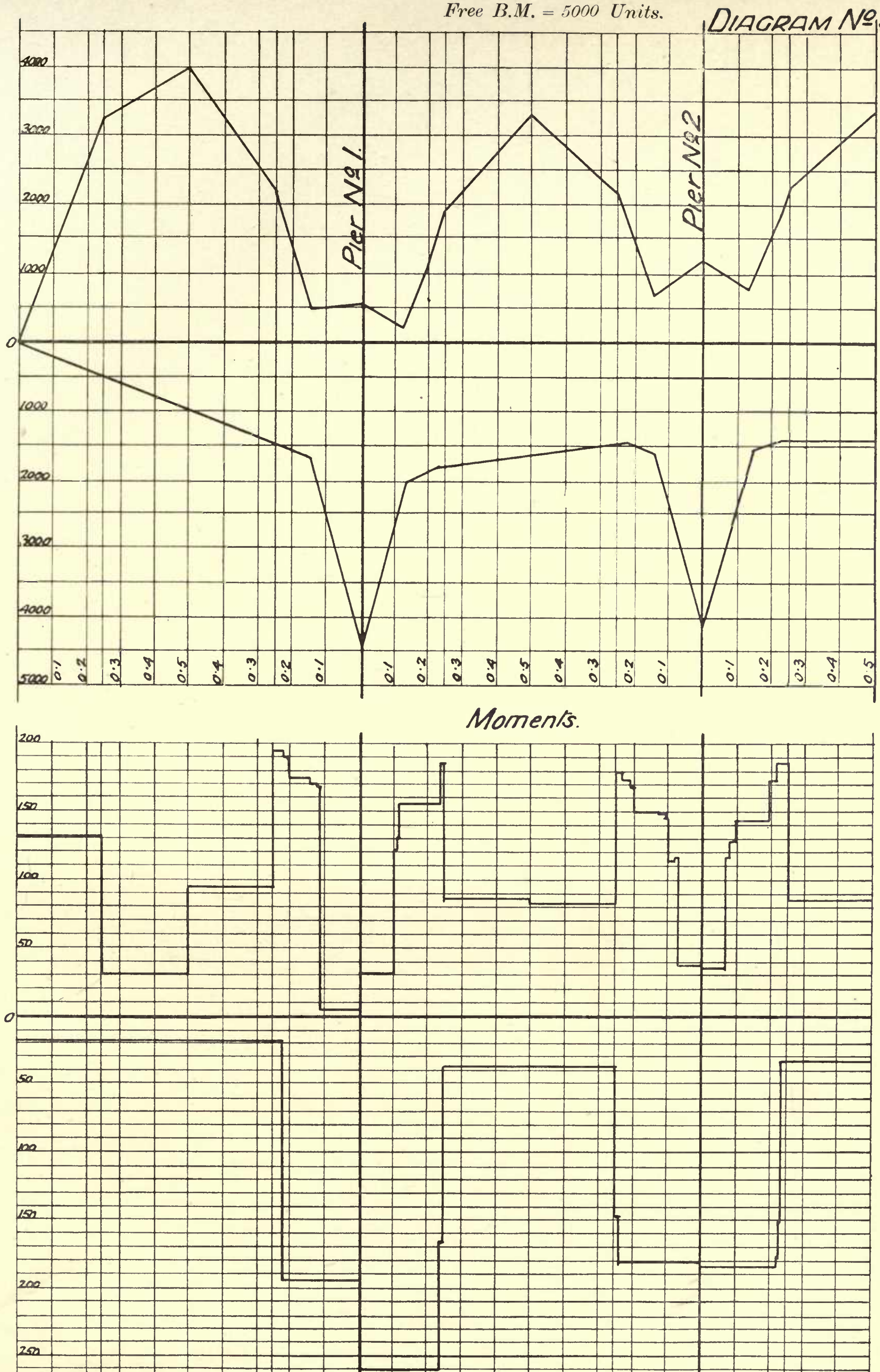


FIVE SPANS.
Free B. M. = 2500 Units.
Moments.
Shears.
Pier №1
Pier №2
2000
1000
0
1000
2000
50
0
50

DIAGRAM No 51.



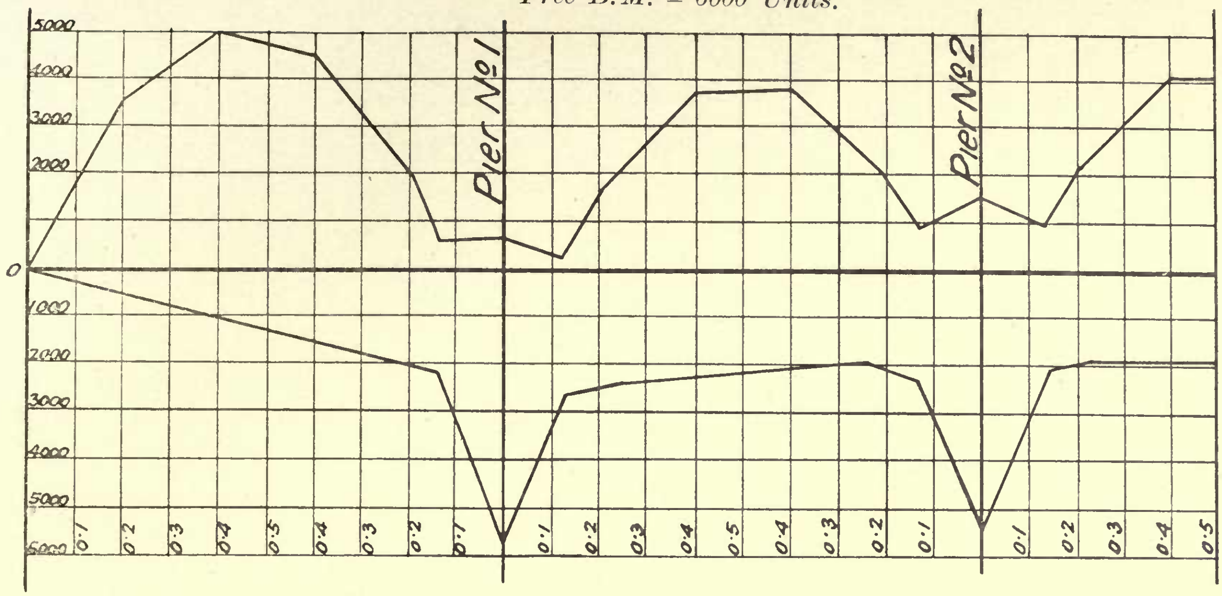
Free B.M. = 5000 Units. DIAGRAM No. 52



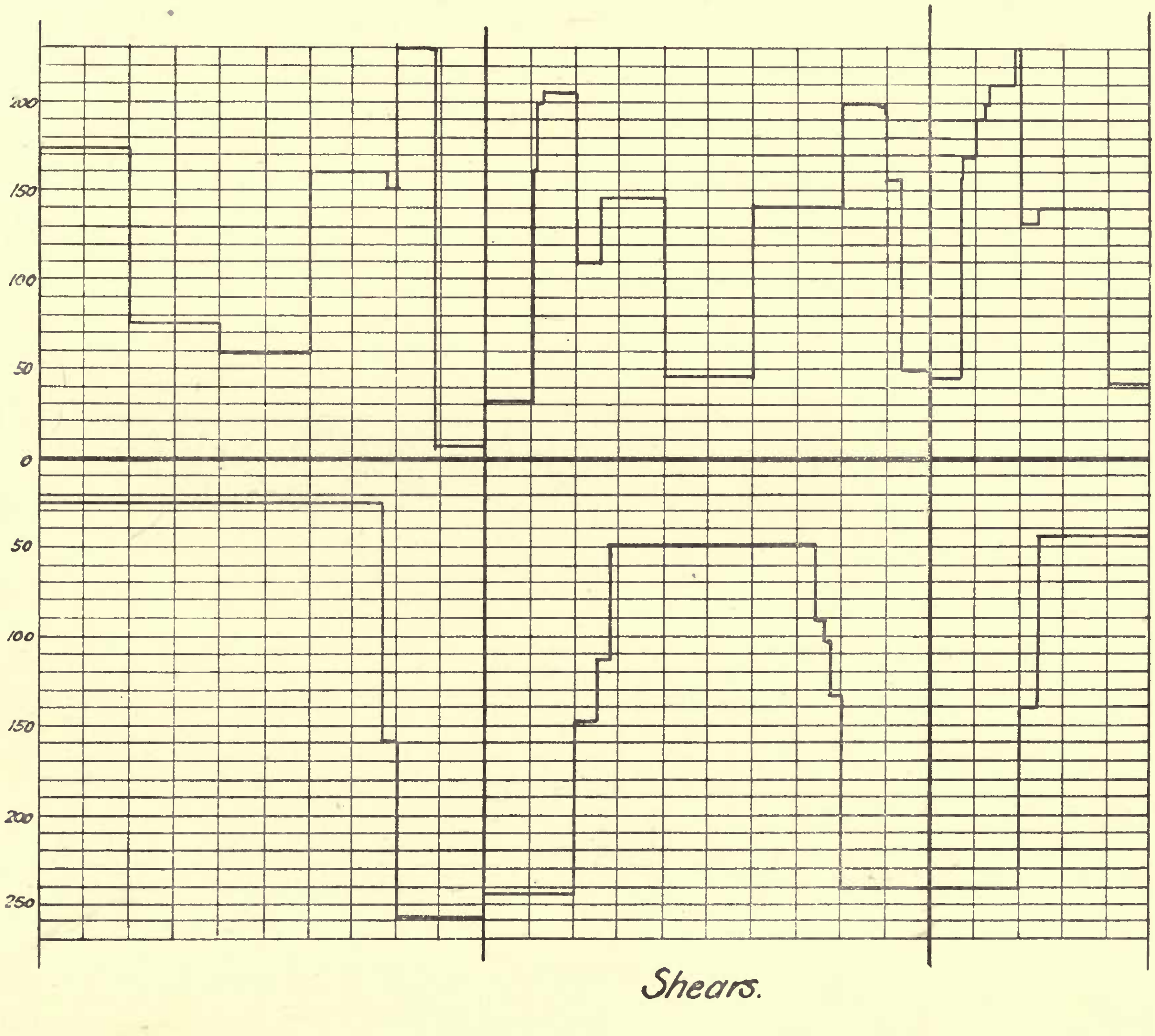
Shears.

DIAGRAM №53.

Free B.M. = 6000 Units.

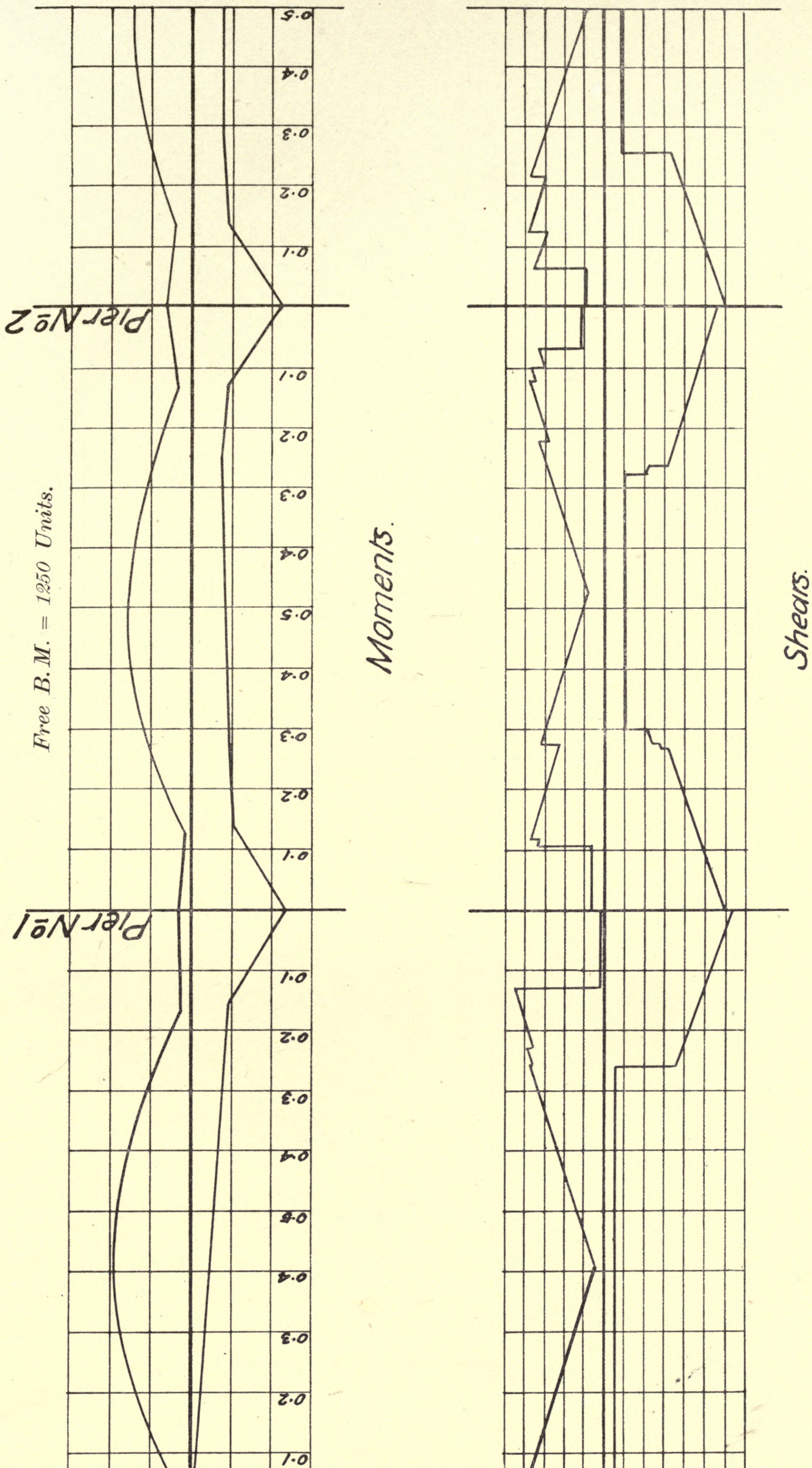


Moments.



Shears.

DIAGRAM No. 54



CHAPTER V.

REACTIONS.

IN connection with the use of the tables 2–8, it will be seen that the loads put upon the columns or supports from main beams, and upon main beams from one secondary beam, may exceed the actual amount of load upon any one span of main, or secondary beam, as the case may be, if all the supports are assumed rigid.

After taking the limiting bending moments and shears for any particular span and load, it will be necessary to multiply the results by certain multiples as given in tables 2–5 and 12–14, since the bending moments in the beam are a function of the load upon the beam. The amounts of these multiples vary with the type of loading, and are derived from that position or positions of loading which give the maximum average reactions from any two adjacent spans, the dead load and live load being considered separately.

LIVE LOADS.

Considering now the conditions of loading necessary to produce the maximum reactions from the live load, and taking for example the Case E (Chapter I.), the following cases occur :—

No. of Spans.	*Maximum Reaction at Outer Supports.*	*Maximum Reaction at Pier No. 1.*	*Maximum Reaction at Pier No. 2.*
Two	Spans loaded 1	Spans loaded 1 & 2	Spans loaded – – –
Three	do do 1 & 3	do do 2 & 3	do do – – –
Five	do do 1 & 3 or 1, 3, 4, & 5	do do 1, 2, 4, & 5	do do 1, 3, & 4

It will be readily seen from the foregoing that it would be almost impossible to have all the floors loaded at the same time with the same peculiar irregularity of live load ; and the condition is certainly not one which need be allowed for in designing the columns of a building with several floors, except perhaps in the case of a building having two spans only in cross section, with two spans of beams longitudinally, when the full increased reactions should be taken into account for the building fully loaded. When we consider the beams, however, both main and secondary, the question becomes more complicated, as will be seen from the following examples, in all of which examples the loading is taken as being that given in Case E, Chap. I.

MULTIPLES FOR REACTIONS—FREE ENDS.

Reactions are expressed in terms of percentage of the full load on one span in Cases A and F, and in percentages of one point load in the remaining cases, B, C, D, and E.

Two Spans. Table No. 2.

CASE.	LOAD.	OUTER SUPPORT.	PIER 1.
A	Dead load	0·375	1·25
B	One point load	0·405	1·38
C	Two ,, ,,	0·837	2·66
D	Three ,, ,,	1·286	4
E	Four ,, ,,	1·4	5·2
F	Distributed ,,	0·437	1·25

Three Spans. Table No. 3.

CASE.	LOAD.	OUTER SUPPORT.	PIER 1.
A	Dead load	0·4	1·1
B	One point load	0·425	1·315
C	Two ,, ,,	0·895	2·52
D	Three ,, ,,	1·315	3·75
E	Four ,, ,,	1·755	4·92
F	Distributed ,,	0·445	1·20

Five Spans. Table No. 4.

CASE.	LOAD.	OUTER SUPPORT.	PIER 1.	PIER 2.
A	Dead load	0·383	1·135	0·98
B	One point load	0·415	1·32	1·25
C	Two ,, ,,	0·856	2·64	2·43
D	Three ,, ,,	1·31	4·51	3·67
E	Four ,, ,,	1·75	5·04	4·75
F	Distributed ,,	0·435	1·235	1·16

Example 1.—First of all consider the main beams, in the case of two spans of secondary beams spanning across the cross section of a building, with a central row of main beams of two equal spans, running along the building at right angles to the secondary beams, each bay being loaded with four secondary beams.

In this case it is possible to increase the load coming upon the secondary beams from the slabs, with all spans loaded, by about 13 per cent. on the end row of secondary beams, the remaining rows of secondary beams have reactions slightly less or not more than the original 100 per cent.

The fact that in the end spans of the outer row of the main beams it is possible to make the loading unequal, due to the end row of secondary beams being more heavily loaded than the inner rows, causes the assumption of regular and even loading on any span to be in error. This is important perhaps in the case of main beams loaded with one point load ; but may be ignored in other cases.

With both spans of secondary beams loaded over the whole cross section, the reaction on the main beams is increased by 25 per cent.

The central column load should therefore be multiplied by $1·25 \times 1·3$, or a total of 1·62 ; a very serious increase over the ordinary assumption of equally distributed load with freely supported spans, in this particular case.*

It has been shewn that it is possible to have cases in which this increment of reaction is progressive in effect, and the amount varies with the particular arrangement of beams. As soon as we have a greater number of spans, we shall find that the increase is smaller in amount.

Example 2.—Consider next the case of a building similar in cross section to the last, but having an extra span of main beams running along the building ; or three spans of main beams altogether.

Firstly, it is possible to increase the reactions from the slabs on to the end row of secondary beams by about 13 per cent., as before.

The maximum reaction from the secondary beams on to the main beam is obtained by loading the full cross section, the increase being 25 per cent.

If we consider the two end bays of main beams to be fully loaded, that is to say with five bays of slab at each end across the full cross section, it will be found that this condition of loading gives the maximum + B.M. in the centre of the outer span, and also the maximum −B.M. in the centre of the centre span.

If we consider the building with two-thirds of the length loaded across the full cross section, it will be found that this produces the maximum −B.M. at the pier in the main beams.

It is therefore necessary in designing the main beams to increase the load to the extent of 25 per cent.

We shall find that the last-mentioned condition of loading produces the maximum reactions on the columns, and in designing the columns therefore, the load should be multiplied by $1·25 \times 1·25$, or 1·537 ; again a very serious increase over the ordinary distribution of load for free spans.

Example 3.—Consider next the same cross section of building, but with two more bays of main beams, making in all five spans of main beams.

* This figure of 1·3 is obtained from Table No. 2 by taking the figure of 5·2 for Case E, and dividing by 4, since there are 4 point loads on one span.

We shall now find that the maximum reaction in the end columns, with bays 1, 2, 4, and 5 loaded across the full cross section, must be multiplied by $1{\cdot}25 \times 1{\cdot}26$, or $1{\cdot}575$; on the inner columns the maximum is a slightly smaller amount, viz., $1{\cdot}484$.

The excess of load on the main beams must be taken as 25 per cent. as in the last case. The cases which we now have considered so far are all reasonably possible; but as soon as we increase the number of bays in cross section, however, the results are very different.

Example 4.—Let us assume the same arrangement as in the last example, but with a further bay of secondary beams in cross section, making three bays of secondary beams and five bays of main beams.

Considering first the reactions from the secondary beams upon the main beams, the condition of loading to give maximum reactions is for bays 2 and 3 in cross section to be loaded, giving an increase of 21 per cent. of load.

Next consider the conditions of loading upon main beams to obtain maximum reactions upon columns at the ends.

A reasonable case to allow for, is with bays 2 and 3 in cross section, and bays 1 and 2 of main beams loaded: when the load on the end columns must be multiplied by $1{\cdot}503$.

In designing the secondary beams they should be considered as having no increment; and in designing the main beams it is reasonable to allow for 21 per cent. increased load.

Example 5.—If we now consider a building with the floors arranged on the same system as in the last example, but with 5 or more bays of secondary beams in cross section, and with 5 or more bays of main beams, and with 16 or more columns, we have far less serious increases.

In order to obtain the maximum reactions upon the main beams, it is necessary to load bays 2, 4, and 5 in cross section, or load bays 1, 2, 4, and 5, which give nearly the same resulting amount of reaction.

The condition required to produce maximum reaction of main beams upon the columns is also with bays 1, 2, 4, and 5 (longitudinally) loaded.

This requirement produces such a peculiarly elaborate system of loading as to be quite beyond the limits of reasonable probability, and need not be allowed for.

The case of the two adjacent bays 1 and 2 of the secondary beams loaded is, however, a reasonable one, and gives 21 per cent. increase of reaction upon the outer main beams.

While this increase of reactions may reasonably be used for the purpose of calculating the beams, it should not be used for calculating the columns, since these critical cases of loading are hardly likely to occur at the same time; but what does appear reasonable, however, is to assume all five spans of main beams loaded with 21 per cent. excess, and the maximum B.M. is then found to be less than the possible loading as shewn in the diagrams 40–54, by about 9 per cent., and it would be sufficient, therefore, if the bending moments in the main beams were designed from the Table No. 8, after multiplying the load by $1{\cdot}21 \times 0{\cdot}91$, or a total of $1{\cdot}10$. With the same conditions of all five spans of main beams loaded, the maximum loads on the outer row of columns become as shewn in Table 5. The loads on the inner row of columns are again for the condition of all 5 spans loaded longitudinally, but bays 2 and 3 only in cross section.

GENERAL REMARKS.—LIVE LOAD.

In the opinion of the author, the question of the increase of the reactions is one which should always be carefully considered for every building; and certainly in the case of Examples 1–5, as described, should be allowed for.

In choosing the live loads to be allowed on a structure, it is usual to make them large enough to cover contingencies; and the individual designer must settle for himself whether it is necessary to make an extra allowance on the loads to cover the increase of the reactions; this allowance certainly being advisable in many cases.

With any number of spans above 5, in either direction, it does not seem to be necessary to allow for any appreciable increase.

As far as the columns are concerned, it does not seem to be necessary to consider any increase in the live loads if the structure consists of three or more floors; but in the case of one floor it is essential, and the allowance might be decreased with the number of floors, except, perhaps, in the case of Example 1, where it would be reasonable to take the full increase due to all the floors being loaded.

DEAD LOAD.

The foregoing remarks apply only to the live load, and the results are very considerably modified in most cases by the effect of the dead load. The neutralising effect of the dead load is dependent upon the proportion of dead to live load in any particular case.

In certain cases there will an increase from the slabs on to the secondary beams, and from the secondary beams on to the main beams, and from the main beams on to the columns; all these increases being progressive in effect. Referring to the columns we shall find that the dead load is increased on all floors; and the corrected reaction, therefore, should be applied to the ordinary distribution of load for free spans, no matter what the number of floors may be.

Summing up the loads to allow, we arrive at the multiples in Table V., which should be applied to the ordinary distribution of loads for free spans, to obtain the designing loads.

FOOTINGS.

When considering the size of the footings, and the permissible pressure on the ground, and the danger and seriousness of any appreciable settlement, increased loads should be allowed, or decreased ground pressure, whichever is preferred.

GENERAL REMARKS ON REACTIONS.

It may reasonably be assumed that the increase in column loads is met by the increase in strength of the concrete with age, before the time when the building is in full use and liable to the loading assumed in the Table 5.—that is to say, provided about six months at least has elapsed after placing the concrete in position.

In the opinion of the author, specifications should give different unit loads per square foot, for the calculation of slabs, secondary beams, main beams, columns, and footings; these variations being on somewhat similar lines to the figures now put forward, and determined for each particular building.

Referring to the secondary beams in Examples 1, 2, and 3, the maximum reactions on the main beams occur at the same time as the maximum B. M. in the secondary beams; and for this reason the end row of secondary beams might reasonably be taken as carrying increased loads, as shewn, provided that the slab is freely supported at the ends of the outer spans. If the ends of the slabs are fixed then this allowance will not be necessary.

When, however, as in Example 4, the maximum B. M. in the secondary beams does not occur at the same time as the maximum reaction on the main beams, there does not seem to be any necessity to increase the loads on the end row of secondary beams.

This remark applies also to Example 5.

The examples discussed have all been with one particular arrangement of beams; but with different arrangements of beams these results will be slightly altered, and it is an easy matter to effect the required alteration.

The question of possible increases of reactions has been discussed at considerable length, as, in the opinion of the author, they are, under certain conditions, quite possible without sensible error, and should be allowed for, when the supports are assumed rigid, and the beams are sufficiently stiff to justify this assumption. When the supports of any beam or column are less stiff than assumed there will, of course, be smaller increases than those given; but as long as there is any negative moment at all at the support there will always be increase of reaction.

The assumption of free ends to the end spans will give results which are on the safe side so far as the intermediate reactions are concerned; the effect of fixing the outer ends being to transfer more load on to the outer supports, and to reduce the loads on the intermediate supports.

TABLES GIVING BENDING MOMENTS AND SHEARS.

The following tables, Nos. 6–8, have been compiled from the diagrams 40–54, and give the bending moments and shearing forces at equal distances along each beam for all the assumed cases of loading; and for two, three, five, or more equal spans, with free ends.

The bending moments are expressed in units in terms of the percentage of the free bending moment which would be induced in a similar span, freely supported at both ends, and subject to the same loading. This is referred to hereafter as the 'free bending moment.'

Table No. 5.

Example.	Number of Floors.	Nature of Load.	B.Ms. Secondary Beams.		B.Ms. Main Beams.		Loads on Columns from Outer Row of Main Beams.		Loads on Columns from Inner Row of Main Beams.	
			Inner Rows.	Outer Rows.	Inner Rows.	Outer Rows.	Intermed.	End.	Intermed.	End.
1.	1	Dead	1·0	1·13	—	1·25	—	1·56	—	—
		Live	1·0	1·13	—	1·25	—	1·62	—	—
	2	Dead	1·0	1·13	—	1·25	—	1·56	—	—
		Live	1·0	1·13	—	1·25	—	1·62	—	—
	3 or more	Dead	1·0	1·13	—	1·25	—	1·56	—	—
		Live	1·0	1·13	—	1·25	—	1·62	—	—
2.	1	Dead	1·0	1·13	—	1·25	—	1·39	—	—
		Live	1·0	1·13	—	1·25	—	1·54	—	—
	2	Dead	1·0	1·13	—	1·25	—	1·39	—	—
		Live	1·0	1·13	—	1·25	—	1·27*	—	—
	3 or more	Dead	1·0	1·13	—	1·25	—	1·39	—	—
		Live	1·0	1·13	—	1·25	—	1·18*	—	—
3.	1	Dead	1·0	1·13	—	1·25	1·23	1·42	—	—
		Live	1·0	1·13	—	1·25	1·48	1·57	—	—
	2	Dead	1·0	1·13	—	1·25	1·23	1·42	—	—
		Live	1·0	1·13	—	1·25	1·24*	1·28*	—	—
	3 or more	Dead	1·0	1·13	—	1·25	1·23	1·42	—	—
		Live	1·0	1·13	—	1·25	1·16*	1·19*	—	—
4.	1	Dead	1·0	1·0	—	1·11	1·09	1·26	—	—
		Live	1·0	1·0	—	1·21	1·44	1·5	—	—
	2	Dead	1·0	1·0	—	1·11	1·09	1·26	—	—
		Live	1·0	1·0	—	1·21	1·22*	1·25*	—	—
	3 or more	Dead	1·0	1·0	—	1·11	1·09	1·26	—	—
		Live	1·0	1·0	—	1·21	1·0 *	1·16*	—	—
5.	1	Dead	1·0	1·0	1·0	1·13	1·11	1·29	0·97	1·11
		Live	1·0	1·0	1·0	1.1	1·16	1·41	1·09	1·31
	2	Dead	1·0	1·0	1·0	1·13	1·11	1·29	0·97	1·11
		Live	1·0	1·0	1·0	1·1	1·0 *	1·2 *	1·0 *	1·0 *
	3 or more	Dead	1·0	1·0	1·0	1·13	1·11	1·29	0·97	1·11
		Live	1·0	1·0	1·0	1·1	1·0 *	1·0 *	1·0 *	1·0 *

* These figures are arbitrary, and to be decided by the individual designer.

Table

Two Spans.

The Bending Moments are expressed in Terms of Percentages of the Maximum Free

Cases A and F, and in percentages of one point

Point in Span.	Dead Load.				1 Point Load.				2 Point Loads.			
	B.Ms.		Shears.		B.Ms.		Shears.		B.Ms.		Shears.	
Outer Support	0	0	0·375	0	0	0	0·405	0·094	0	0	0·837	0·166
0·1	26	0	0·275	0	17	3·7	0·405	0·094	25	5	0·837	0·166
0·15	36	0	0·225	0	25·6	5·6	0·405	0·094	37·5	7·5	0·837	0·166
0·2	44	0	0·175	0	34·1	7·5	0·405	0·094	50	10	0·837	0·166
0·25	50	0	0·125	0	42·6	9·4	0·405	0·094	62·5	12·5	0·837	0·166
0·3	54	0	0·075	0	51·1	11·2	0·405	0·094	75	15	0·837	0·166
0·33	—	—	—	—	—	—	—	—	83·3	16·5	0·837	0·166
0·35	56	0	0·025	0	59·6	13·1	0·405	0·094	82·5	17·5	0·333	0·166
0·4	56	0	0·025	0	68·2	15·0	0·405	0·094	80	20	0·333	0·166
0·45	54	0	0·075	0	76·7	16·8	0·405	0·094	77·5	22·5	0·333	0·166
0·5	50	0	0·125	0	85·2	18·7	0·69	0·094	75	25	0·333	0·166
0·45	44	0	0·175	0	73	20·6	0·69	0·094	72·5	27·5	0·333	0·166
0·4	36	0	0·225	0	60·9	22·4	0·69	0·094	70	30	0·333	0·166
0·35	26	0	0·275	0	48·6	24·3	0·69	0·094	67·5	32·5	0·333	0·166
0·33	—	—	—	—	—	—	—	—	66·7	33·5	1·333	0·166
0·3	14	0	0·325	0	36·5	26·2	0·69	0·69	54·6	35	1·333	0·166
0·25	0	0	0·375	0·375	24·3	28·1	0·69	0·69	36·4	37·5	1·333	1·333
0·2	0	20	0	0·425	12·2	29·9	0·595	0·69	18·2	40	1·333	1·333
0·15	0	40	0	0·475	0	31·8	0·595	0·69	0	42·5	1·333	1·333
0·1	0	60	0	0·525	0	46·2	0	0·69	0	61·7	0	1·333
Pier No. 1	0	100	0	0·625	0	75	0	0·69	0	100	0	1·333
	+	−	+	−	+	−	+	−	+	−	+	−
	Case A.				Case B.				Case C.			

No. 6.
Free Ends.

Bending Moments ; and the Shears in percentages of the total load on one span in load in the remaining Cases B, C, D, and E.

3 Point Loads.				*4 Point Loads.*				*Distributed Load.*			
B.Ms.		*Shears.*		*B.Ms.*		*Shears.*		*B.Ms.*		*Shears.*	
0	0	1·286	0·234	0	0	1·716	0·3	0	0	0·438	0·063
25·3	4·8	1·286	0·234	28·4	5·1	1·716	0·3	31	5	0·34	0·063
38	7·2	1·286	0·234	42·5	7·6	1·716	0·3	43·5	7·5	0·293	0·063
50·7	9·6	1·286	0·234	56·7	10·2	1·716	0·3	54	10	0·244	0·063
63·3	12	1·286	0·234	62·5	12·7	0·716	0·3	62·5	12·5	0·196	0·063
66	14·4	0·286	0·234	68·4	15·3	0·716	0·3	69	15	0·147	0·063
—	—	—	—	—	—	—	—	—	—	—	—
68·7	16·8	0·286	0·234	74·2	17·8	0·716	0·3	73·5	17·5	0·099	0·063
71·3	19·2	0·286	0·234	80	20·4	0·716	0·3	76	20	0·05	0·063
73·9	21·6	0·286	0·234	77·5	22·9	0·6	0·3	76·5	22·5	0·096	0·063
76·6	24·1	1·0	0·234	75	25·5	0·6	0·3	75	25	0·143	0·063
69·2	26·5	1·0	0·234	72·5	28	0·6	0·3	71·5	27·5	0·189	0·063
61·9	28·9	1·0	0·234	70	30·6	1·6	0·3	66	30	0·235	0·063
54·5	31·3	1·0	0·234	59·1	33·1	1·6	0·3	58·5	32·5	0·281	0·063
—	—	—	—	—	—	—	—	—	—	—	—
47·2	33·8	1·0	0·234	48·3	35·7	1·6	0·3	49	35	0·328	0·063
39·8	36·2	2·0	2·0	37·5	38·2	1·6	1·6	37·5	37·5	0·375	0·375
19·9	38·6	2·0	2·0	26·7	40·8	2·284	2·6	24	40	0·393	0·425
0	41	1·714	2·0	8·9	43·3	2·284	2·6	8·5	42·5	0·438	0·475
0	58·6	1·714	2·0	0	62·2	2·284	2·6	0	54	0	0·525
0	93·8	0	2·0	0	100	0	2·6	0	100	0	0·625
+	−	+	−	+	−	+	−	+	−	+	−
Case D.				*Case E.*				*Case F.*			

Table
Three Spans.

The Bending Moments are expressed in terms of percentages of the Maximum Free
Cases A and F ; and in percentages of one point

Point in Span.	*Dead Load.*				*1 Point Load.*				*2 Point Loads.*			
	B. Ms.		*Shears.*		*B. Ms.*		*Shears.*		*B. Ms.*		*Shears.*	
Outer Support	0	0	0·39	0	0	0	0·425	0·075	0	0	0·87	0·133
0·1	28	0	0·293	0	17	2·9	0·425	0·075	26·3	3·9	0·87	0·133
0·15	39	0	0·244	0	25·5	4·4	0·425	0·075	39·4	5·9	0·87	0·133
0·2	48	0	0·195	0	34	5·8	0·425	0·075	52·6	7·8	0·87	0·133
0·25	55	0	0·146	0	42·5	7·5	0·425	0·075	65·7	9·8	0·87	0·133
0·3	60	0	0·099	0	51	8·7	0·425	0·075	78·8	11·8	0·87	0·133
0·33	—	—	—	—	—	—	—	—	86·7	12·9	0·87	0·133
0·35	63	0	0·05	0	59·5	10·4	0·425	0·075	86	13·7	0·31	0·133
0·4	64	0	0	0	68	11·7	0·425	0·075	84	15·7	0·31	0·133
0·45	63	0	0·055	0	76·5	13·1	0·425	0·075	82	17·7	0·31	0·133
0·5	60	0	0·11	0	85	14·6	0·675	0·075	80	19·6	0·31	0·133
0·45	55	0	0·166	0	73	16	0·675	0·075	78	21·6	0·31	0·133
0·4	48	0	0·221	0	62	17·5	0·675	0·075	76	23·5	0·31	0·133
0·35	39	0	0·276	0	51	19	0·675	0·075	74·0	25·5	0·31	0·133
0·33	—	—	—	—	—	—	—	—	73·3	26·3	1·31	0·133
0·3	28	0	0·331	0	39	20·4	0·675	0·675	62·9	27·5	1·31	0·133
0·25	15	0	0·39	0·39	28	21·9	0·675	0·675	45·7	29·4	1·31	1·31
0·2	0	8·9	0	0·415	17	23·3	0·64	0·675	28·5	31·4	1·31	1·31
0·15	0	26·6	0	0·463	8·5	24·8	0·6	0·675	11·4	33·4	1·15	1·31
0·1	0	44·3	0	0·512	9·0	39·9	0·58	0·675	12	53·3	1·15	1·31
Pier No. 1	0	80	0	0·61	10	70	0·025	0·675	13·3	93·3	0·044	1·31
Pier No. 1	0	80	0	0·50	10	70	0·125	0·645	13·3	93·3	0·177	1·213
0·1	0	53·3	0	0·403	5	43·3	0·36	0·645	8·4	59·5	1·04	1·213
0·15	0	40	0	0·355	2·5	30	0·5	0·645	6	42·7	1·04	1·213
0·2	0	26·7	0	0·307	10	30	0·5	0·645	21	40	1·04	1·213
0·25	0	13·3	0	0·258	20	30	0·645	0·645	36	40	1·213	1·213
0·3	4	0	0·21	0·21	30	30	0·645	0·645	51	40	1·213	0·177
0·33	—	—	—	—	—	—	—	—	60	40	1·213	0·177
0·35	11	0	0·157	0	40	30	0·645	0·125	60	40	0·213	0·177
0·4	16	0	0·105	0	50	30	0·645	0·125	60	40	0·213	0·177
0·45	19	0	0·525	0	60	30	0·645	0·125	60	40	0·213	0·177
0·5	20	0	0	0	70	30	0·645	0·125	60	40	0·213	0·177
	+	−	+	−	+	−	+	−	+	−	+	−
	Case A.				*Case B.*				*Case C.*			

No. 7.
Free Ends.

Bending Moment; and the Shears in percentages of the total load on one Span in load in the remaining Cases B, C, D, and E.

3 Point Loads.				*4 Point Loads.*				*Distributed Load.*			
B.Ms.		*Shears.*		*B.Ms.*		*Shears.*		*B.Ms.*		*Shears.*	
0	0	1·315	0·188	0	0	1·755	0·24	0	0	0·445	0·05
26·2	3·8	1·315	0·188	29·3	4	1·755	0·24	32	4	0·445	0·05
39·3	5·7	1·315	0·188	44	6	1·755	0·24	45	6	0·346	0·05
52·4	7·6	1·315	0·188	58·7	8·1	1·755	0·24	56	8	0·297	0·05
65·6	9·5	1·315	0·188	65	10·1	0·755	0·24	65	10	0·247	0·05
68·7	11·4	0·315	0·188	71·3	12·1	0·755	0·24	72	12	0·148	0·05
—	—	—	—	—	—	—	—	—	—	—	—
71·8	13·3	0·315	0·188	77·7	14·1	0·755	0·24	77	14	0·1	0·05
74·9	15·2	0·315	c·188	84	16·2	0·755	0·24	80	16	0·05	0·05
78	17·1	0·315	0·188	82	18·2	0·56	0·24	80	18	0·096	0·05
81·2	19	0·93	0·188	80	20·2	0·56	0·24	80	20	0·132	0·05
74·3	20·9	0·93	0·188	78	22·2	0·56	0·24	77	22	0·178	0·05
67·5	22·8	0·93	0·188	76	24·3	1·56	0·24	72	24	0·224	0·05
60·6	24·7	0·93	0·188	65·7	26·3	1·56	0·24	65	26	0·27	0·05
—	—	—	—	—	—	—	—	—	—	—	—
53·8	26·6	0·93	0·188	55·3	28·3	1·56	0·24	56	28	0·323	0·05
46·9	28·5	1·93	1·93	45	30·3	1·56	1·56	45	30	0·37	0·37
28·7	30·4	1·93	1·93	34·7	32·3	2·5	2·56	32	32	0·4	0·422
10·6	32·3	1·722	1·93	11·3	34·4	2·5	2·56	17	34	0·425	0·474
11·2	50·7	1·722	1·93	12	54	2·29	2·56	12	53·8	0·445	0·526
12·5	87·5	0·062	1·93	13·3	93·3	0·08	2·56	13·3	93·2	0·017	0·63
12·5	87·5	0·312	1·82	13·3	93·3	0·4	2·36	13·3	93·2	0·083	0·58
6·2	56	1·5	1·82	6·7	53·3	2·0	2·36	6·7	57·7	0·38	0·49
12·5	40·3	1·5	1·82	13·3	43	2·0	2·36	11	40	0·326	0·445
25	38·9	1·82	1·82	26·7	41·3	2·0	2·36	24	40	0·273	0·4
37·5	37·5	1·82	1·82	35	40	1·36	1·30	35	40	0·31	0·355
42·5	37·5	0·82	0·312	43·3	40	1·36	1·0	44	40	0·248	0·31
—	—	—	—	—	—	—	—	—	—	—	—
47·5	37·5	0·82	0·312	51·7	40	1·36	0·4	51	40	0·186	0·083
52·5	37·5	0·82	0·312	60	40	1·36	0·4	56	40	0·124	0·083
57·5	37·5	0·82	0·312	60	40	0·36	0·4	59	40	0·062	0·083
62·5	37·5	0·82	0·312	60	40	0·36	0·4	60	40	0	0·083
+	−	+	−	+	−	+	−	+	−	+	−
Case D.				*Case E.*				*Case F.*			

Table No. 8.

Five Spans. Free Ends.

The Bending Moments are expressed in terms of percentages of the Maximum Free Bending Moment ; and the Shears in percentages of the total load on one span in Cases A and F, and in percentages of one point load in the remaining Cases B, C, D, and E.

Point in Span.	Dead Load.				1 Point Load.				2 Point Loads.				3 Point Loads.				4 Point Loads.				Distributed Loads.			
	B.Ms.		Shears.		B.Ms.		Shears.		B.Ms.		Shears.		B.Ms.		Shears.		B.Ms.		Shears.		B.Ms.		Shears.	
Outer Support	0	0	0·383	0	0	0	0·42	0·079	0	0	0·857	0·14	0	0	1·31	0·197	0	0	1·735	0·253	0	0	0·438	0·053
0·1	30·2	0	0·287	0	16·8	3·1	0·42	0·079	26	4·2	0·857	0·14	26	4	1·31	0·197	29	4·2	1·735	0·253	31·7	4·2	0·341	0·053
0·15	41·9	0	0·239	0	35·2	4·6	0·42	0·079	39	6·3	0·857	0·14	39	6	1·31	0·197	43·5	6·2	1·735	0·253	44·5	6·3	0·293	0·053
0·2	51·2	0	0·192	0	33·6	6·2	0·42	0·079	52	8·4	0·857	0·14	52	8	1·31	0·197	58·1	8·3	1·735	0·253	59·4	8·5	0·244	0·053
0·25	58	0	0·144	0	42	7·7	0·42	0·079	65	10·5	0·857	0·14	65	10	1·31	0·197	64·2	10·4	0·735	0·253	64·2	10·7	0·196	0·053
0·3	62·8	0	0·096	0	50·4	9·2	0·42	0·079	78	12·6	0·857	0·14	68	12	0·31	0·197	70·3	12·5	0·735	0·253	71·1	12·8	0·147	0·053
0·33	—	—	—	—	—	—	—	—	85·5	13·9	0·857	0·14	—	—	—	—	—	—	—	—	—	—	—	—
0·35	64·9	0	0·048	0	58·8	10·8	0·42	0·079	84·8	14·7	0·317	0·14	71	14	0·31	0·197	76·5	14·6	0·735	0·253	76	14·9	0·09	0·053
0·4	65	0	0	0	67·2	12·3	0·42	0·079	82·7	16·8	0·317	0·14	74	16	0·31	0·197	82·6	16·6	0·735	0·253	78·8	17	0·05	0·053
0·45	62·8	0	0·077	0	75·6	13·9	0·42	0·079	80·6	18·9	0·317	0·14	77	18	0·31	0·197	79·6	18·7	0·58	0·253	79·6	19·1	0·09	0·053
0·5	58	0	0·128	0	84	15·4	0·68	0·079	78·5	21	0·317	0·14	80	20	0·95	0·197	76·7	20·8	0·58	0·253	78·5	21·3	0·143	0·053
0·45	51·1	0	0·179	0	73	16·9	0·68	0·079	76·4	23·1	0·317	0·14	73	22	0·95	0·197	73·8	22·9	0·58	0·253	75·3	23·4	0·189	0·053
0·4	41·8	0	0·230	0	62	18·3	0·68	0·079	74·3	25·2	0·317	0·14	66	24	0·95	0·197	70·9	25	1·58	0·253	70·2	25·5	0·236	0·053
0·35	30·2	0	0·281	0	51	20·0	0·68	0·079	72·2	27·3	0·317	0·14	59	26	0·95	0·197	61·3	27	1·58	0·253	63	27·6	0·282	0·053
0·33	—	—	—	—	—	—	—	—	71·4	28·2	1·317	0.14	—	—	—	—	—	—	—	—	—	—	—	—
0·3	16·3	0	0·332	0	40	21·5	0·68	0·079	61·1	29·4	1·317	0·14	52	28	0·95	0·197	51·6	29·1	1·58	0·253	54	29·7	0·328	0·053
0·25	0	0	0·383	0·383	29	23·1	0·68	0·68	44	31·5	1·317	1·317	44·9	30	1·95	1·95	41·9	31·2	1·58	1·68	42·8	31·8	0·375	0·365
0·2	0	16·8	0	0·43	18	24·6	0·658	0·68	26·9	33·6	1·317	1·317	27	32	1·95	1·95	32·3	33·3	2·29	2·58	29·6	33·9	0·375	0·419
0·15	0	33·6	0	0·477	7	26·2	0·6	0·68	9·8	35·7	1·12	1·317	9·2	33·9	1·75	1·95	9·8	35·4	2·29	2·58	14·5	36	0·438	0·473
0·1	0	50·4	0	0·524	7·5	41·4	0·02	0·68	10·4	55·9	1·12	1·317	9·7	52·5	1·75	1·95	10·4	55·5	2·29	2·58	10·4	55·9	0·438	0·527
Pier No. 1	0	84	0	0·617	8·6	71·8	0.02	0·68	11·5	96	0·038	1·317	10·8	89·7	0·054	1·95	11·5	95·7	0·069	2·58	11·6	95·6	0·014	0·635

Pier No. 1	0	84	0	0·517	8·6	71·8	0·107	0·643	11·5	96	0·191	1·263	10·8	89·7	0·319	2·61	11·5	95·7	0·315	2·46	11·6	95·6	0·071	0·608
0·1	0	56	0	0·418	4·3	45·3	0·107	0·643	4·5	61·8	1·03	1·263	16	57·1	1·58	2·61	5·8	60·9	2·055	2·46	5·8	61	0·375	0·515
0.15	0	42	0	0·369	2·2	32	0·59	0·643	5·1	44·7	1·03	1·263	20	40·8	1·58	2·61	13·4	43·5	2·055	2·46	10·6	43·7	0·35	0·468
0·2	0	28	0	0·319	10	31·3	0·59	0·643	20·7	40	1·263	1·263	29	37	1·85	2·61	26·8	35·5	2·055	2·46	24·1	42·2	0·3	0·421
0·25	0	14	0	0·27	20·4	30·5	0·643	0·643	36·4	39·3	1·263	1·263	38	35·7	0·85	2·61	35·6	35·2	1·46	1·48	35·6	40·7	0·325	0·375
0·3	0	0	0·22	0·22	30·8	29·8	0·643	0·643	52	38·5	1·263	1·263	43·5	35·1	0·85	0·385	44·4	34·9	1·46	1·15	45·2	39·2	0·3	0·328
0·33	—	—	—	—	—	—	—	—	61·4	38·1	1·263	0·274	—	—	—	—	—	—	—	—	—	—	—	—
0·35	10·6	0	0·171	0	41·2	29·1	0·643	0·154	61·6	37·8	0·263	0·274	49	34·7	0·85	0·385	53·3	34·6	1·46	0·492	52·7	38·4	0·256	0·103
0·4	18·5	0	0·122	0	51·6	28·4	0·643	0·154	62·1	37·1	0·263	0·274	54·5	34·1	0·85	0·385	62·1	34·2	1·46	0·492	58·2	37·8	0·212	0·103
0·45	23·8	0	0·073	0	62	27·6	0·643	0·154	62·6	36·4	0·263	0·274	60	33·0	0·85	0·385	62·6	33·9	0·46	0·492	61·7	37	0·168	0·103
0·5	26·5	0	0·024	0	72·4	26·9	0·643	0·154	63·2	35·6	0·263	0·274	65·5	33·1	0·85	0·385	63·1	33·6	0·46	0·492	63·2	36·2	0·124	0·103
0·45	26·5	0	0·024	0	61·8	26·2	0·61	0·154	63·7	34·9	0·263	0·274	61	32·7	0·82	0·385	63·6	33·3	0·46	0·492	62·7	35·4	0·08	0·103
0·4	23·8	0	0·073	0	51·2	25·5	0·61	0·154	64·2	34·2	0·263	0·274	56·5	32·1	0·82	0·385	64·2	33	1·41	0·492	60·2	34·6	0·141	0·103
0·35	18·5	0	0·122	0	40·6	24·7	0·61	0·154	64·8	33·5	0·263	0·274	51·9	31·7	0·82	0·385	56·4	32·6	1·41	0·492	55·8	33·8	0·202	0·103
0·33	—	—	—	—	—	—	—	—	65	33·2	1·2	0·274	—	—	—	—	—	—	—	—	—	—	—	—
0·3	10·6	0	0·171	0	30	24	0·61	0·61	56·2	32·7	1·2	1·2	47·4	31·1	0·82	0·385	48·6	32·3	1·41	0·92	49·3	33	0·263	0·318
0·25	0	0	0·22	0·22	23·3	25·3	0·61	0·61	41·8	32	1·2	1·2	42·9	30·7	1·79	1·82	40·8	32	1·41	1·41	40·8	32·2	0·325	0·359
0·2	0	12·6	0	0·272	16·5	26·6	0·5	0·61	27·4	34·1	1·2	1·2	28·1	30	1·79	1·82	33	35	2·0	2·41	30·4	34·8	0·325	0·4
0·15	0	25·2	0	0·325	9·7	27·8	0·5	0·61	13	36·2	1·02	1·2	13·4	32·2	1·5	1·82	12·9	38	2·0	2·41	17·9	37·4	0·35	0·441
0·1	0	37·8	0	0·377	12·9	40·8	0·5	0·61	17·3	53·8	1·02	1·2	17	49·3	1·5	1·82	17·2	55·1	2·0	2·41	17·2	54·6	0·375	0·482
Pier No. 2	0	63	0	0·483	19·4	66·8	0·154	0·61	25·9	89	0·78	1·2	24·3	83·4	1·05	1·82	25·8	89·4	1·56	2·41	25·8	89	0·325	0·565
Pier No. 2	0	63	0	0·5	19·4	66·8	0·131	0·64	25·9	89	0·8	1·256	24·3	83·4	1·42	1·85	25·8	89·4	1·7	2·41	25·8	89	0·35	0·6
0·1	0	35	0	0·4	12·9	39·4	0·477	0·64	17·3	52·2	1·05	1·256	18·1	49·3	1·42	1·85	17·2	53	2·1	2·41	17·2	53·7	0·375	0·535
0·15	0	21	0	0·35	9·7	25·7	0·523	0·64	13	33·8	1·05	1·256	15	32·2	1·42	1·85	12·9	34·8	2·31	2·41	12·9	36	0·375	0·47
0·2	0	0	0	0·3	18·7	24·7	0·6	0·64	27·5	33·5	1·256	1·256	29·8	30·9	1·85	1·85	35·7	31·8	2·31	2·41	21·8	34·6	0·338	0·405
0·25	11·3	0	0·275	0·275	27·6	23·7	0·64	0·64	42	33·2	1·256	1·256	44·7	29·6	1·85	1·85	43·1	31·6	1·41	1·41	34·7	33·1	0·3	0·34
0·3	20·6	0	0·2	0	36·6	23·7	0·64	0·136	56·5	32·9	1·256	0·243	48·9	29·6	0·85	0·34	50·5	31·6	1·41	0·437	43·7	31·6	0·256	0·09
0·33	—	—	—	—	—	—	—	—	66·2	32·7	1·256	0·243	—	—	—	—	—	—	—	—	—	—	—	—
0·35	27·8	0	0·15	0	45·5	23·7	0·64	0·136	65·9	32·6	0·256	0·243	53	29·6	0·85	0·34	57·9	31·6	1·41	0·437	51·6	31·6	0·212	0·09
0·4	32·9	0	0·1	0	54·5	23·7	0·64	0·136	65	32·3	0·256	0·243	57·2	29·6	0·85	0·34	65·0	31·6	1·41	0·437	57·5	31·6	0·168	0·09
0·45	36	0	0·05	0	63·5	23·7	0·64	0·136	64·1	32	0·256	0·243	61·3	29·6	0·85	0·34	63·1	31·6	0·41	0·437	61·4	31·6	0·124	0·09
0·5	37	0	0	0	72·4	23·7	0·64	0·136	63·2	31·7	0·256	0·243	65·5	29·6	0·85	0·34	61·3	31·6	0·41	0·437	63·3	31·6	0·08	0·09
	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—
	Case A.				*Case B.*				*Case C.*				*Case D.*				*Case E.*				*Case F.*			

T 2

CHAPTER VI.

REFERRING to Tables 6–8, we find by subtraction the difference in bending moment between any two points at regular distances apart throughout the span of the beam, expressed in percentages of the free bending moment at the centre of the span.

The case of the horizontal shear in the beam, which has to be transmitted through the concrete to the tension steel, and the necessary provision for the same, is somewhat similar to the case of a plate girder, where, in order to determine the horizontal shear, we divide the difference in bending moment between any two points in the beam by the effective depth of the girder between the centres of gravity of the flanges, the result being the amount of the horizontal shear which has to be taken up by rivets between those points in transmitting the load from the web to the flanges.

In the case of the concrete beam, having obtained the required percentage difference from the tables, we multiply it by the free bending moment for the given span and load, and we get the required amount of bending moment developed between the two points.

Calling this amount A, and the effective depth of the beam from the centre of gravity of the steel to the centre of compression of the beam d, and the horizontal shear S, we have

$$S = \frac{A}{d}$$

Calling the safe stress in the steel in shear f, and the required area of steel between the two points in the beam, q,

$$q = \frac{S}{f}$$

which gives the required area of steel in the form of stirrups or binding to resist the horizontal shear between the two points.

In the foregoing, no account is taken of the strength of the concrete to resist shear, which under the higher limits of stress is a very doubtful quantity, and should be ignored. If it is required to take into account the strength of the concrete, it should be allowed for as follows :—

Calling the breadth of the beam b,

$$q = \frac{S - 60\,bp}{f}$$

the safe shearing stress in the concrete being taken at 60 lbs. per sq. inch in the formula.

In the case of **T** beams, it should be noted that b in the above formula can only be taken as the width of the rib of the **T** beam and not the width of the flange.*

The resistance of the concrete is usually small ; and for high stresses it is safer to ignore it. Its amount of course is dependent upon whether we are considering tension or compression.

The work done by the concrete is usually so small in comparison with the work done by the stirrups in resisting shear, and the effect of displacement of the stirrups is so serious that it is advisable to space them rather closer than calculation demands.

* This is true when dealing with the difference in tension, or if the Neutral Axis comes below the bottom of the slab. If dealing with difference in compression when the Neutral Axis falls within the slab, then the width of flange of **T** beam can be substituted.

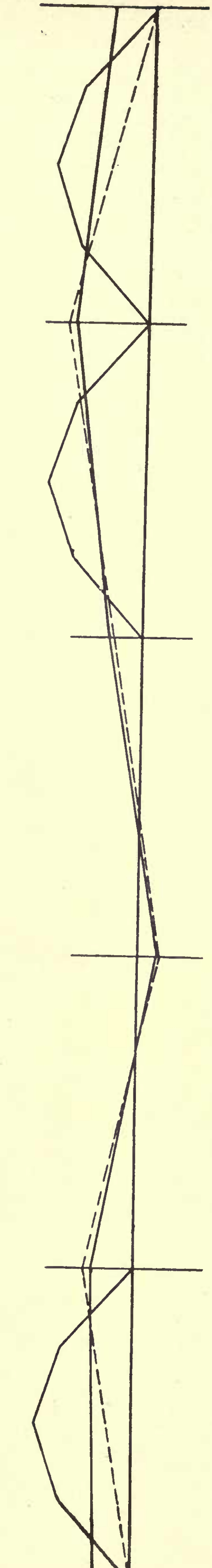


Fig 4.

CHAPTER VII.

IN the case of the ends of the outside spans of a series of continuous beams being fixed, by columns or other means, a negative bending moment is caused in the beam at the outer end, the amount being dependent upon the span and stiffness of the beam, as well as upon the stiffness of the column, &c.

It is not possible to lay down any simple formula having general application to such cases, and as, under any conditions, the reduction in the section of the beam can only be secured at the expense of a corresponding increase in the section of the column, it is questionable whether it is of any advantage to consider a negative bending moment at the outer end due to live loads of greater amount than will reduce the positive moment of the beam in the outer span to about the same amount as the positive moment in the adjoining span. This equals a reduction in the free bending moment of a maximum of about 20% in Case C and a minimum of 15% in Case D.

As far as the dead load in Case A is concerned, this is reduced about 21%.

Except in the case of the dead load, the positive moments in the various spans are approximately equal (within an error of under 5%), and the beams could therefore be kept of the same section throughout, and with equal spans between columns this might be of some practical advantage.

With reference to the negative moments at the piers, there is a reduction of about 10–12% of the moment at the first pier, and a slightly smaller reduction at the centre pier, the difference between the moments at the two piers being very much reduced, the maximum difference being less than 3% for the cases of live load and 9% for dead load.

This would be equivalent to raising the base line of the B.M. diagrams at the outer supports and slightly raising or lowering it at other points; as is shown in Fig. 4, which has the bending moments for free and fixed ends plotted on the same base line, to show the change under one case of loading; the fixed end diagram being shown in full lines, and the other in dotted lines.

The actual amount of the negative bending moment which

we decide to allow at the outer columns being put into the general formula in place of M_1, which has been assumed = 0, the new bending moments, &c., are then calculated.*

In the case of secondary beams, however, it must be borne in mind that in the ends of buildings it frequently happens that some only of the beams run into columns, and the others into main beams spanning between the columns.

The assumption of an appreciable amount of fixedness of ends is not justifiable, therefore, in the case of those running into main beams, owing to the twisting moment put upon the main beam, and is of no great advantage in the case of those running into columns, though in all such cases a certain amount of negative moment should be allowed for whether the positive moment is reduced in consequence or not.

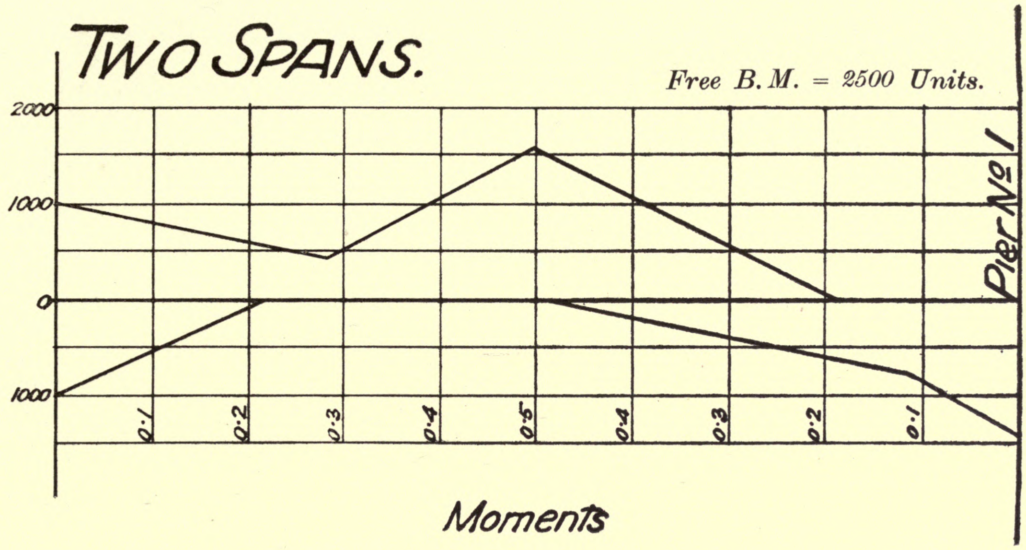
In the chapter on Haunches to Beams it will be seen that the maximum reduction of 20 % of the positive free bending moment corresponds to a negative moment at the ends of the outside beams of about 40 % of the free bending moment which is nearly covered by the bending moment at the commencement of the haunch, and is more than covered by the haunch at the column, and if, in order to keep the same centering as for the inner spans, we were to add similar haunches at the outside columns, we should have provided the necessary resisting moment there, and at the same time improved the conditions of bending moment in the outer columns. This haunch is advisable in all cases, for other reasons.

Speaking generally, therefore, it is not advantageous to take more than 40 % of the free bending moment at the outer ends, and the columns or anchorages must be checked to ascertain whether they are capable of taking up the bending moment induced in them, and must be strengthened if necessary.

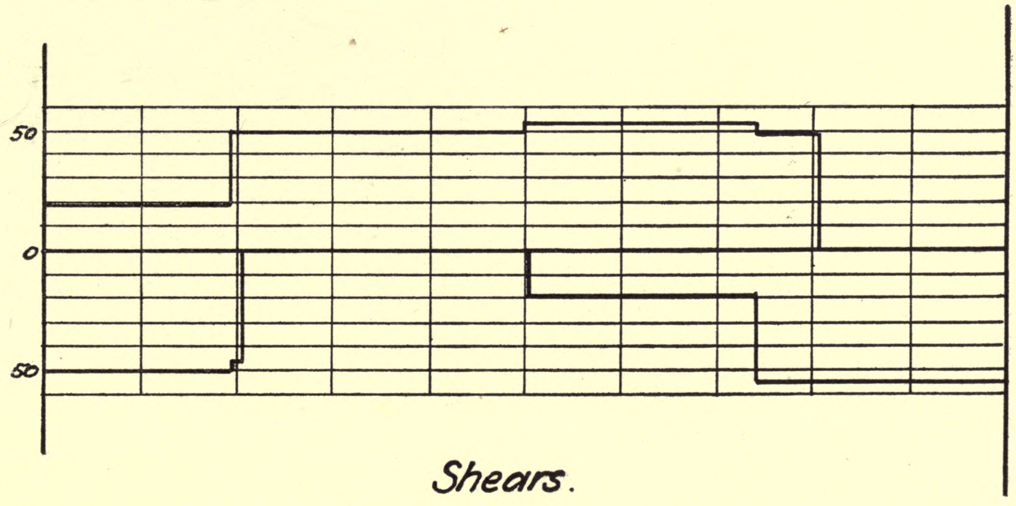
The following diagrams, Nos. 55–69, for two, three, or five spans and upwards, have been compiled from diagrams similar to diagrams 1–39, but with an assumed negative moment round the ends, amounting to 40 % of the free bending moment.

* See pages 3 and 4, and Appendix I.

DIAGRAM Nº 55.

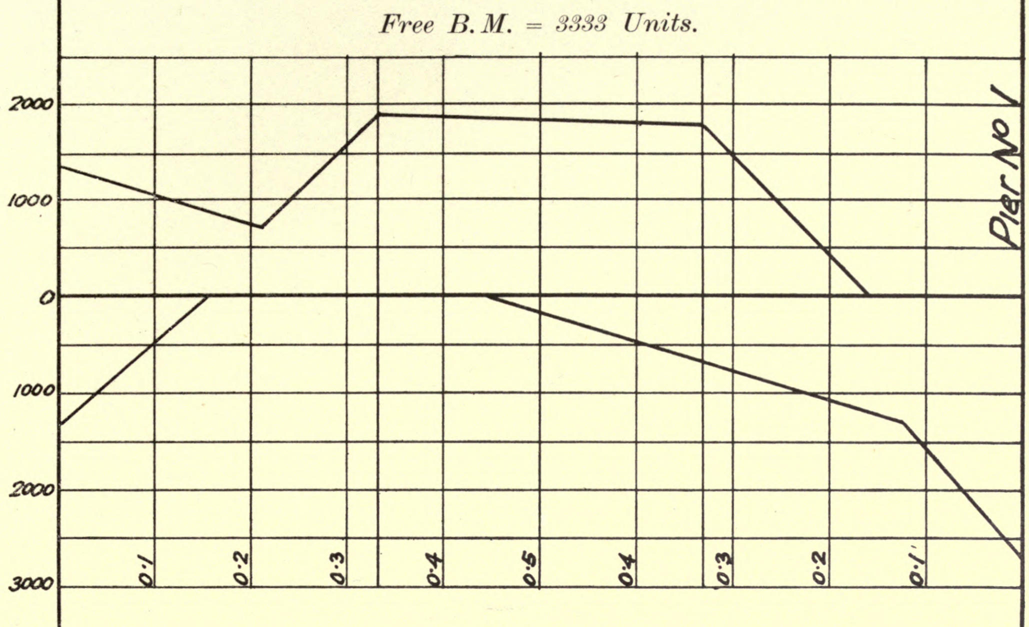


Moments

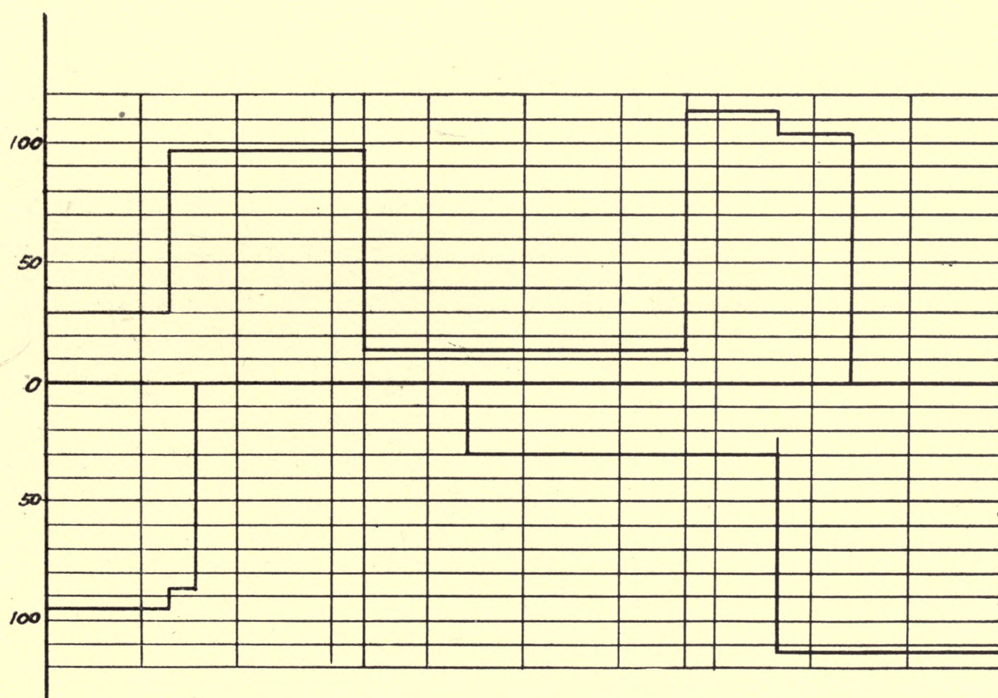


Shears.

DIAGRAM No. 56.



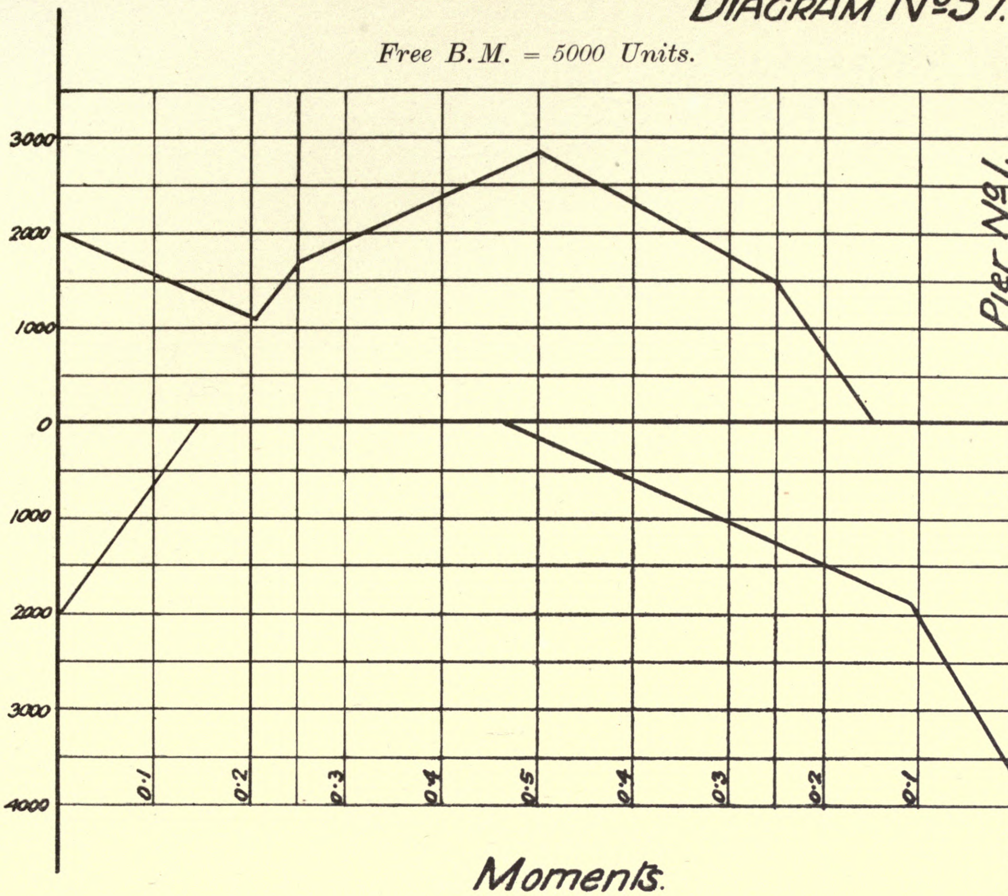
Moments.



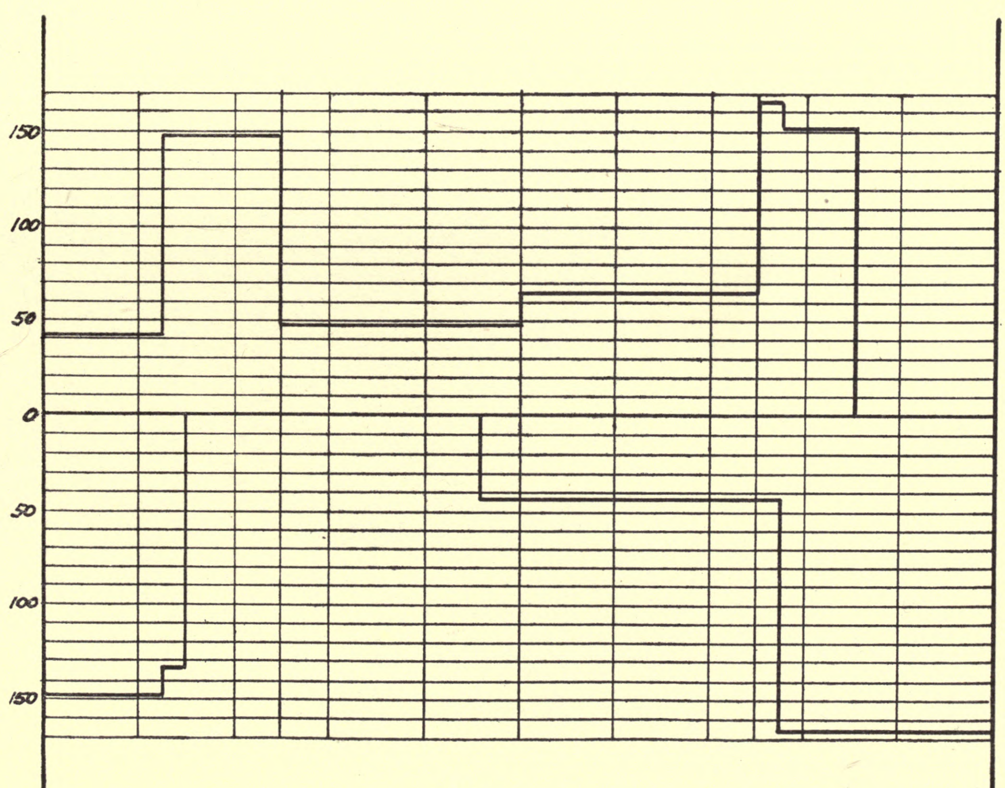
Shears.

DIAGRAM No. 57.

Free B.M. = 5000 Units.



Moments.



Shears.

DIAGRAM Nº 58.

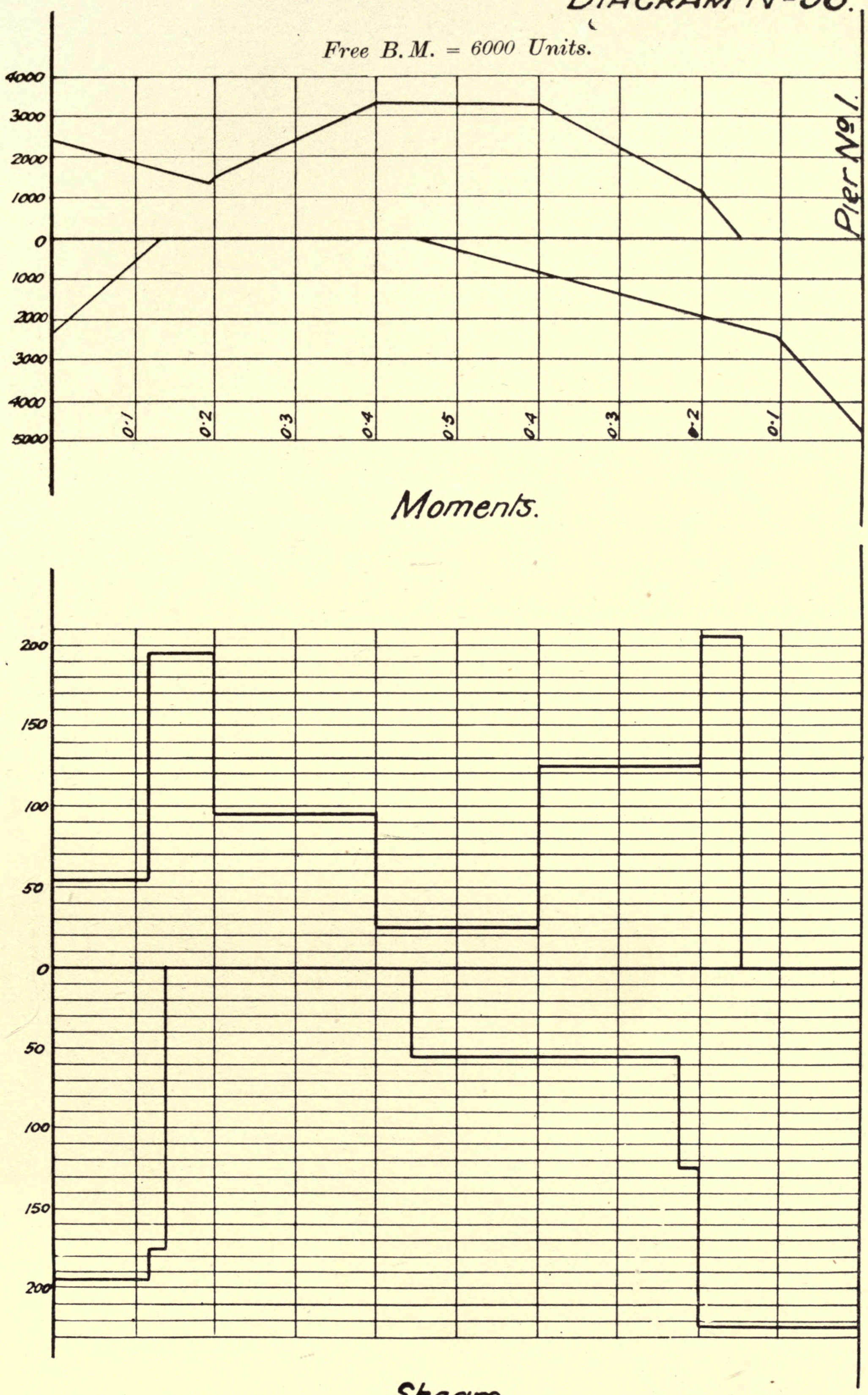
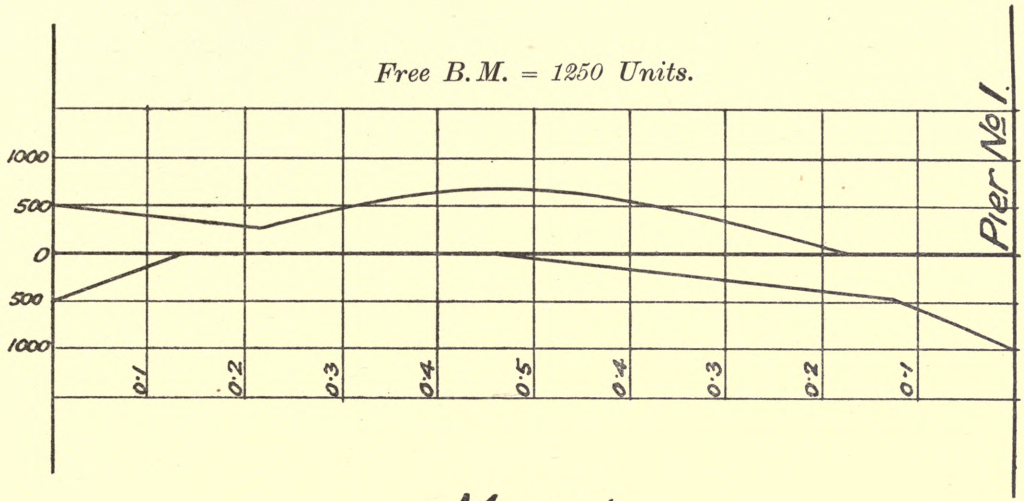
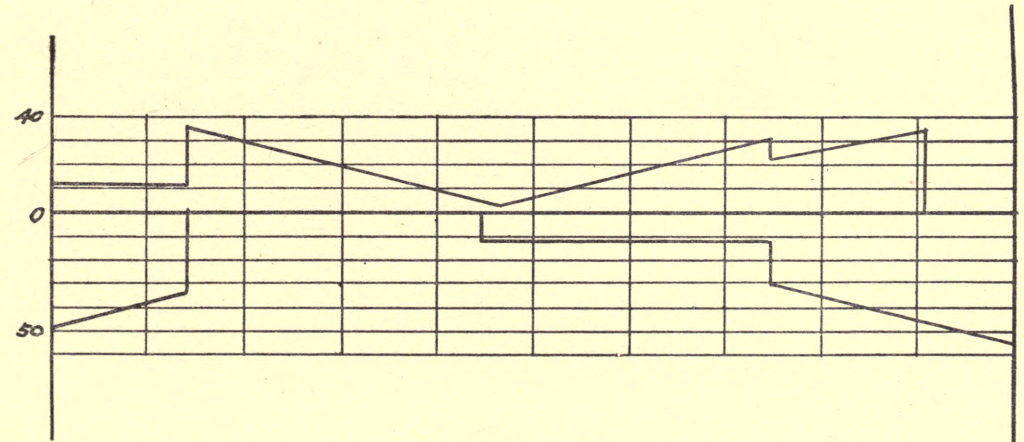


DIAGRAM No. 59.



Moments



Shears.

DIAGRAM No 60.

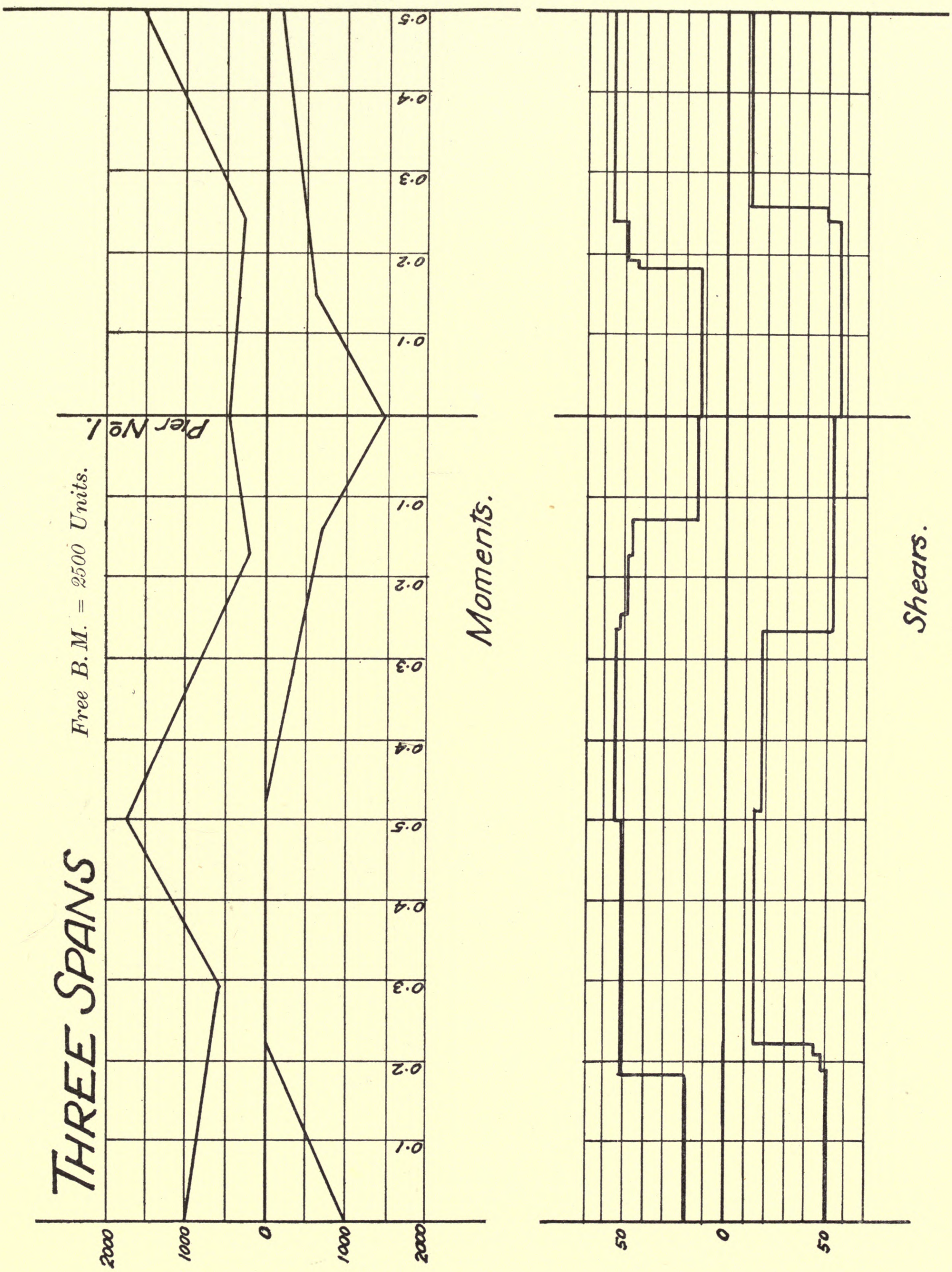


DIAGRAM No. 61.

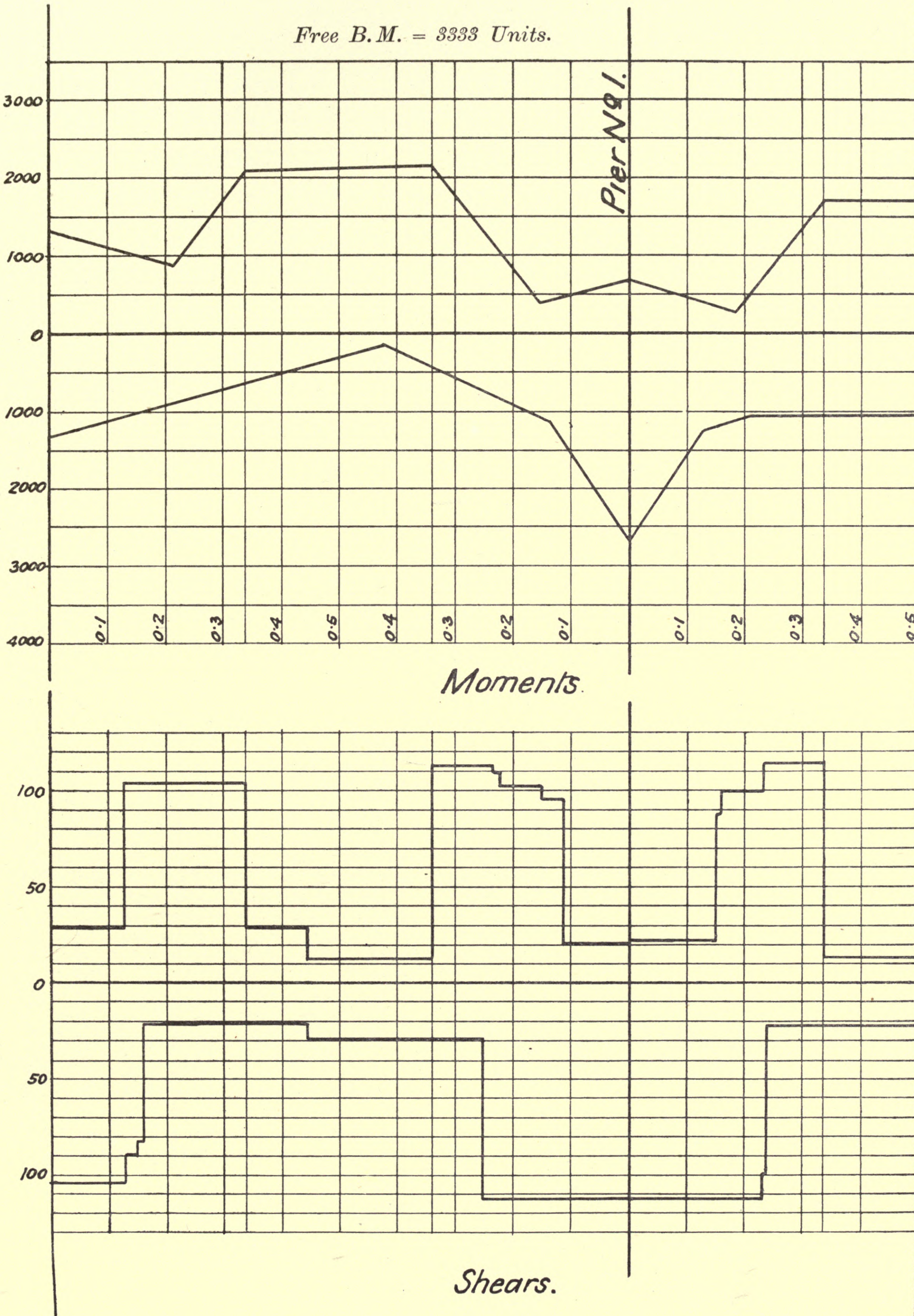


DIAGRAM № 62.

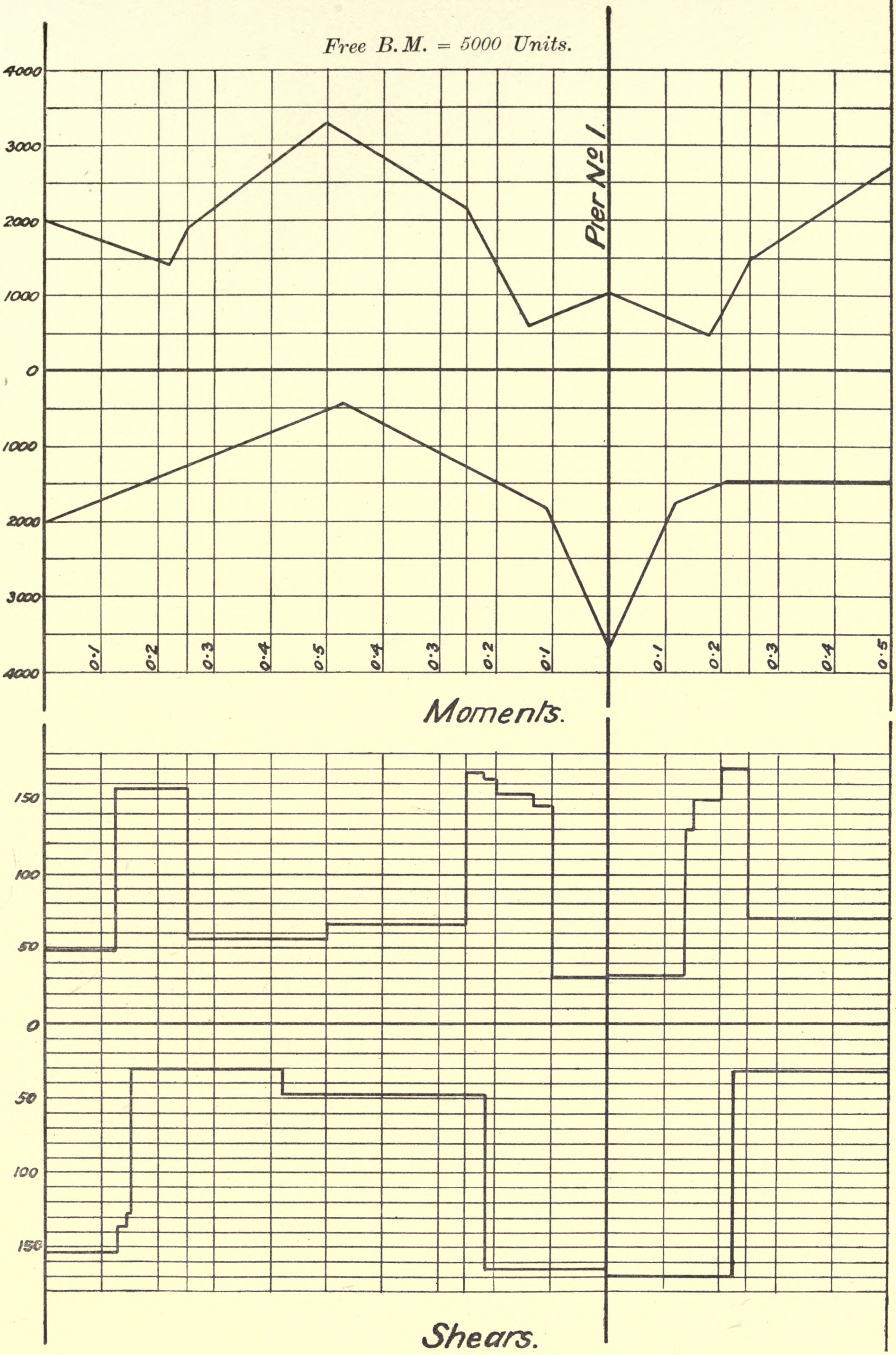
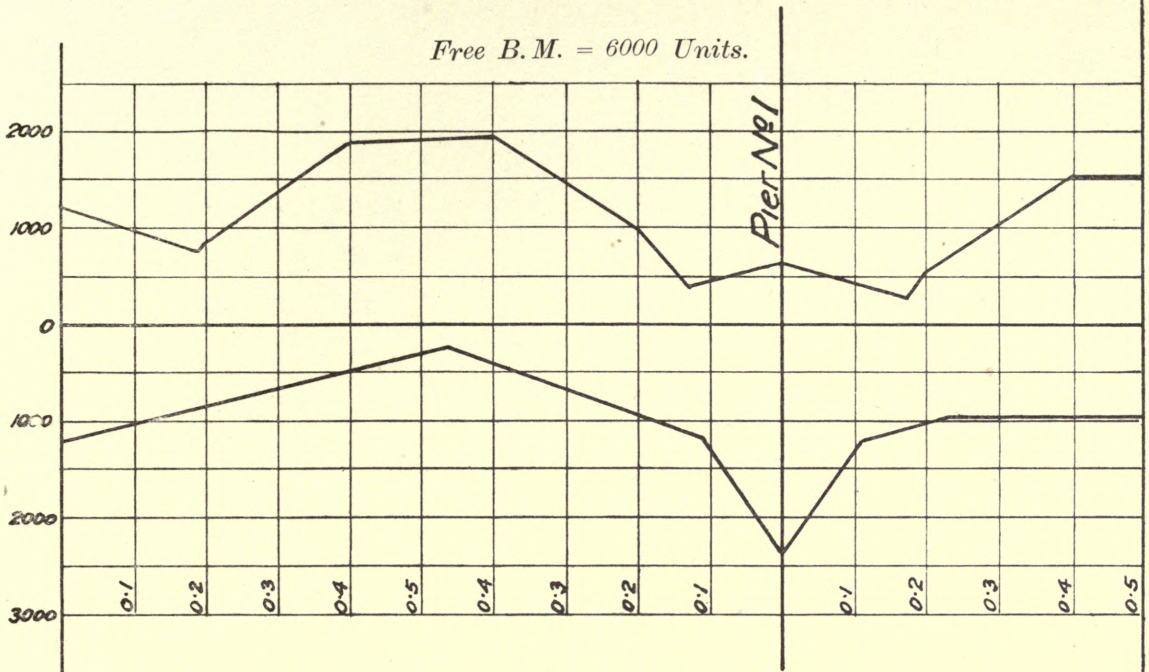
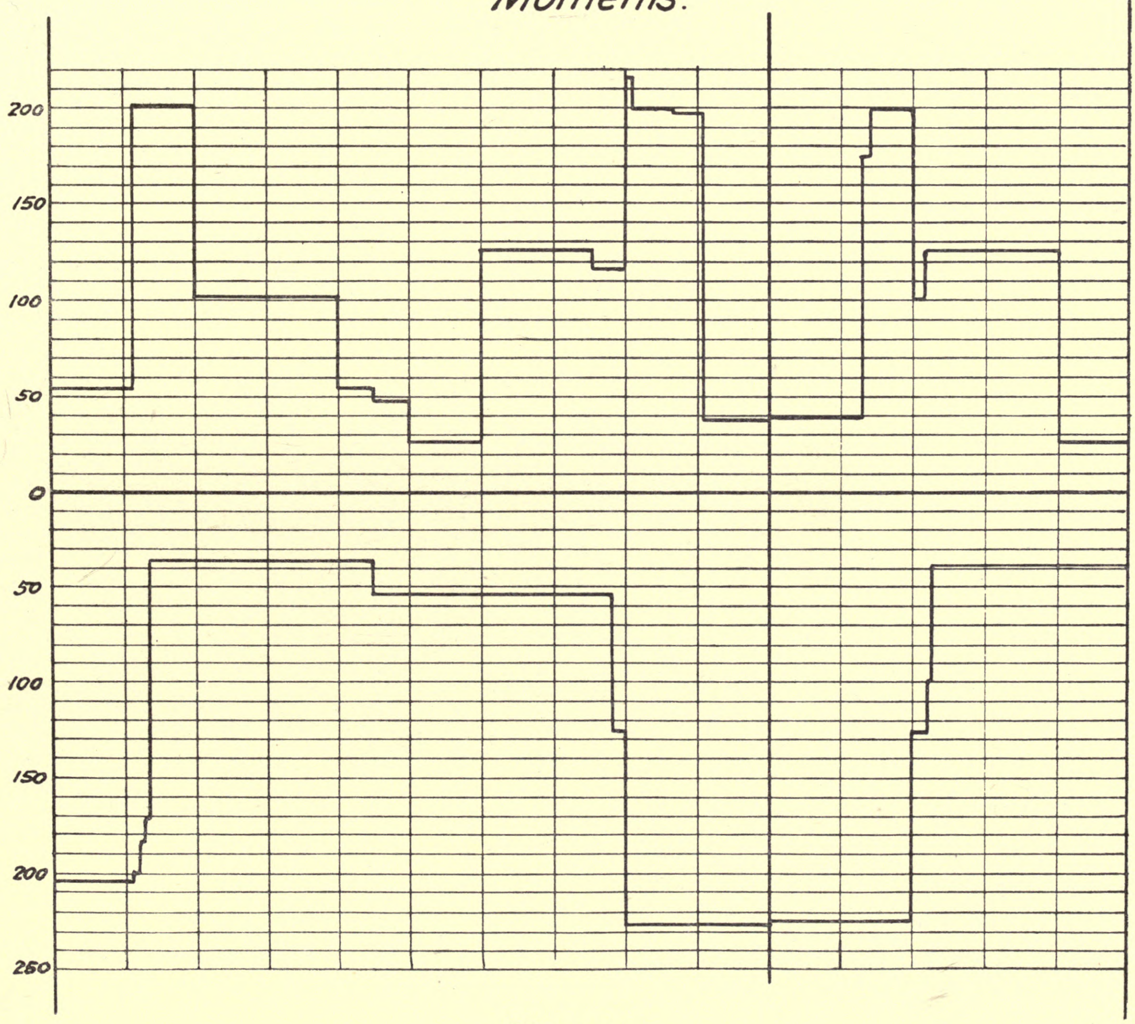

Free B. M. = 5000 Units.
4000
3000
2000
1000
0
1000
2000
3000
4000
Pier № 1
0·1
0·2
0·3
0·4
0·5
0·4
0·3
0·2
0·1
0·1
0·2
0·3
0·4
0·5
Moments.
150
100
50
0
50
100
150
Shears.

Diagram No. 63.



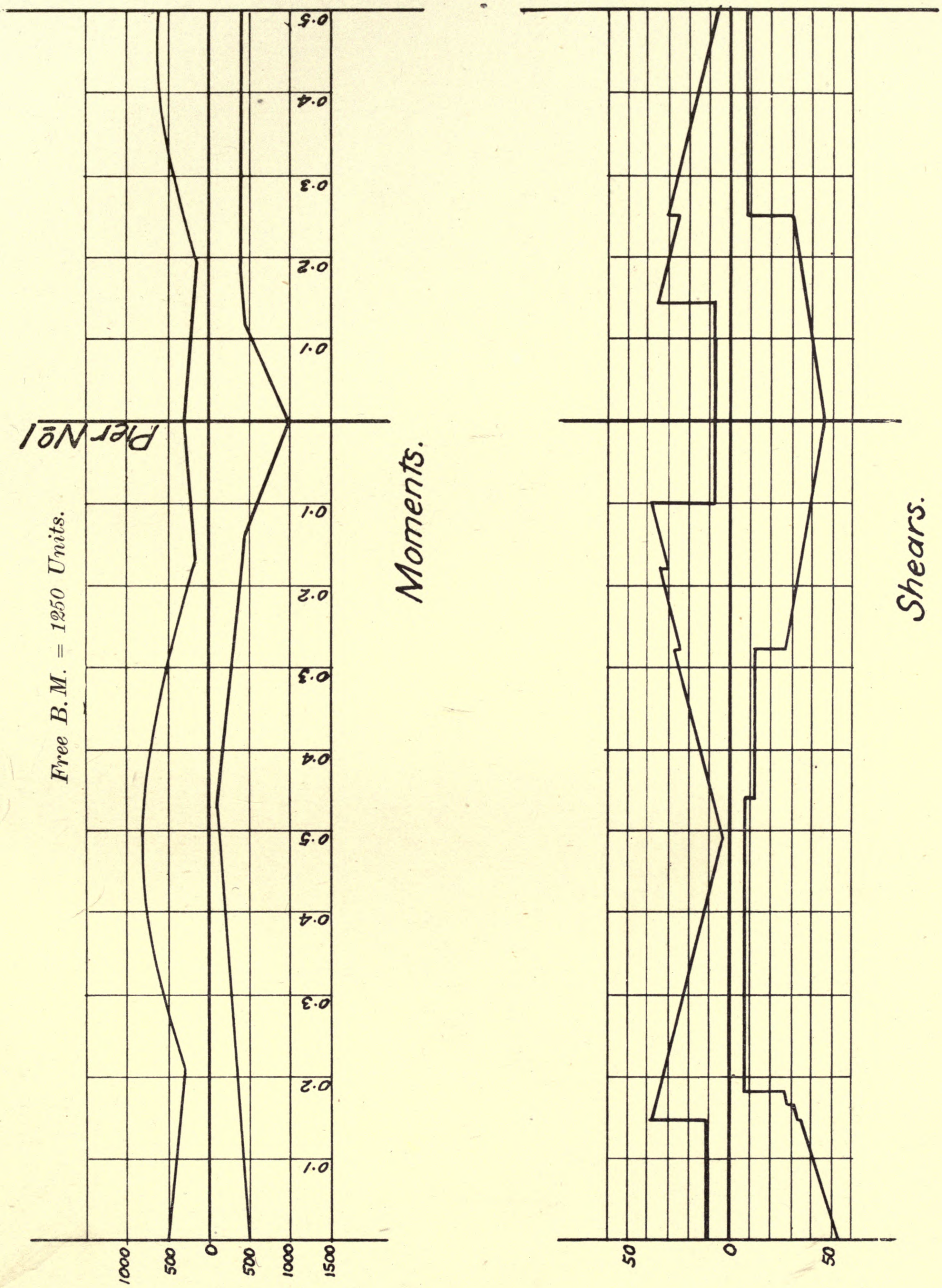
Moments.



Shears.

Y

DIAGRAM Nº 64.



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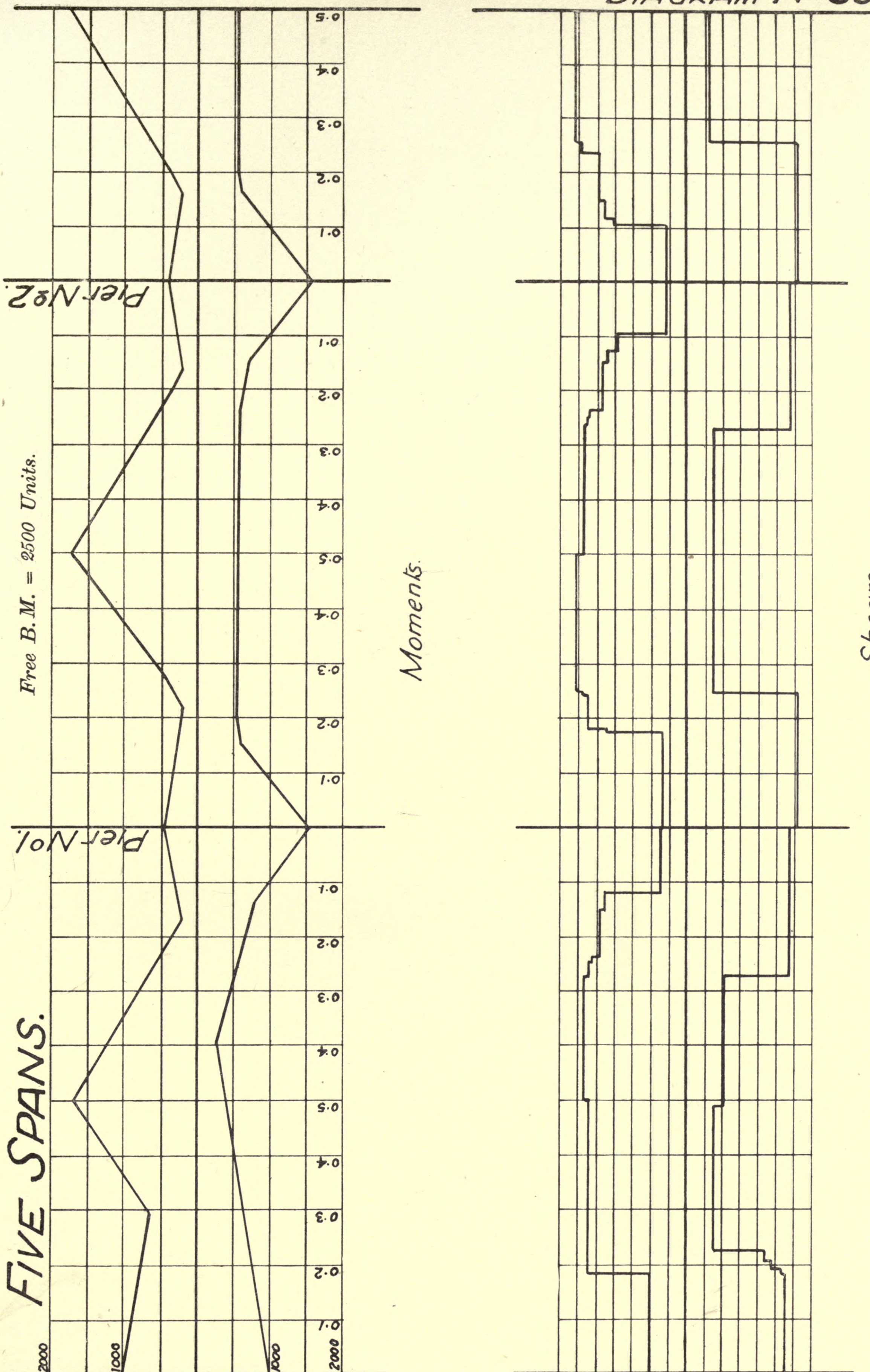
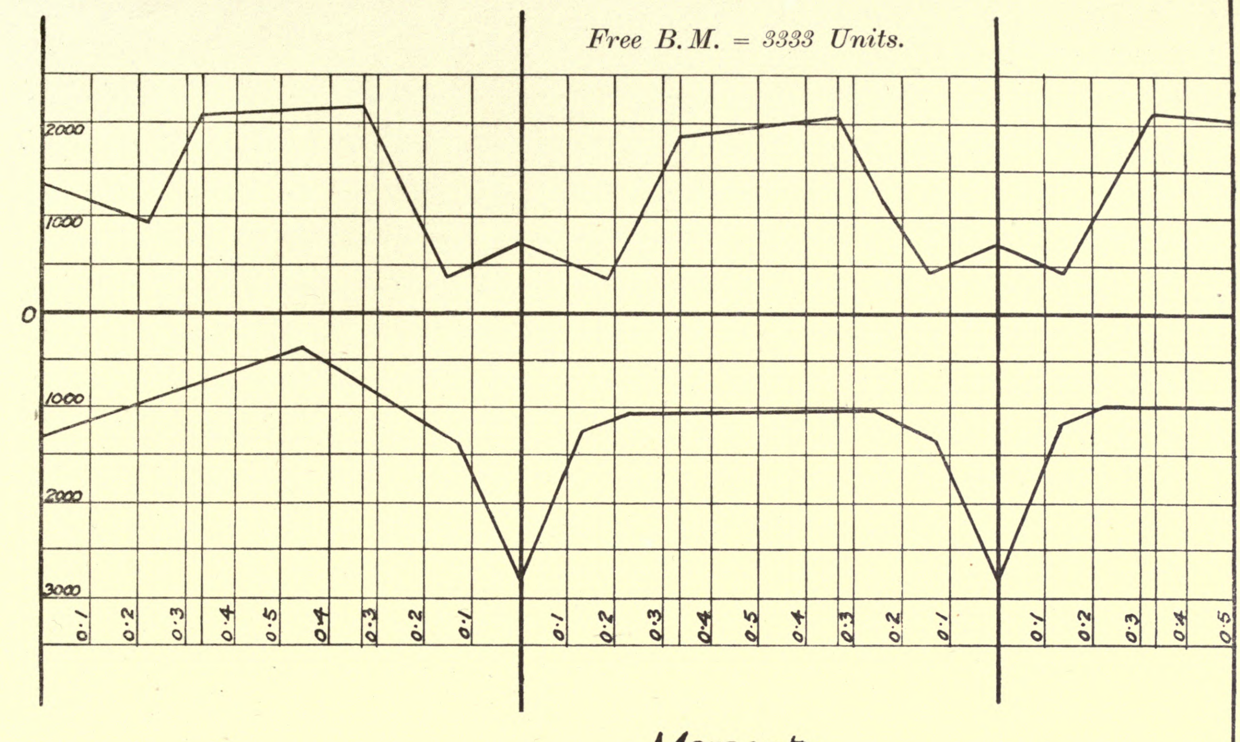
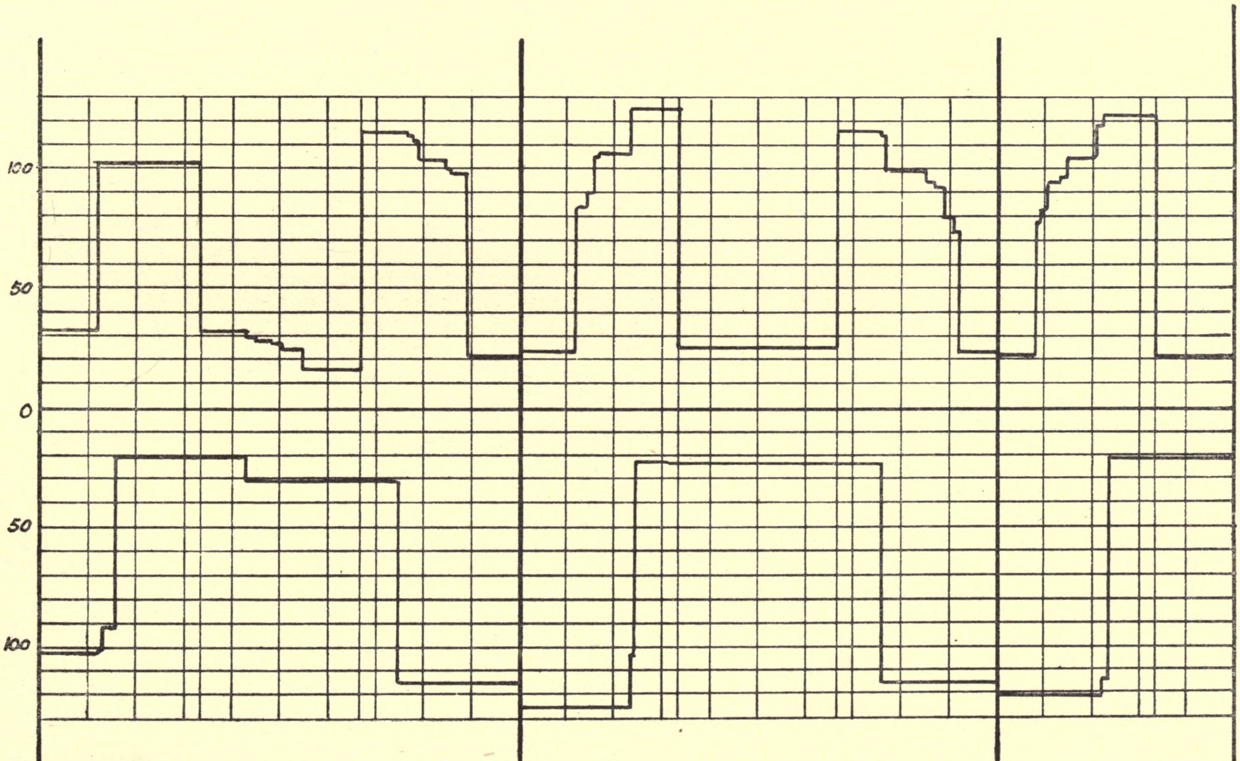


DIAGRAM Nº 66.

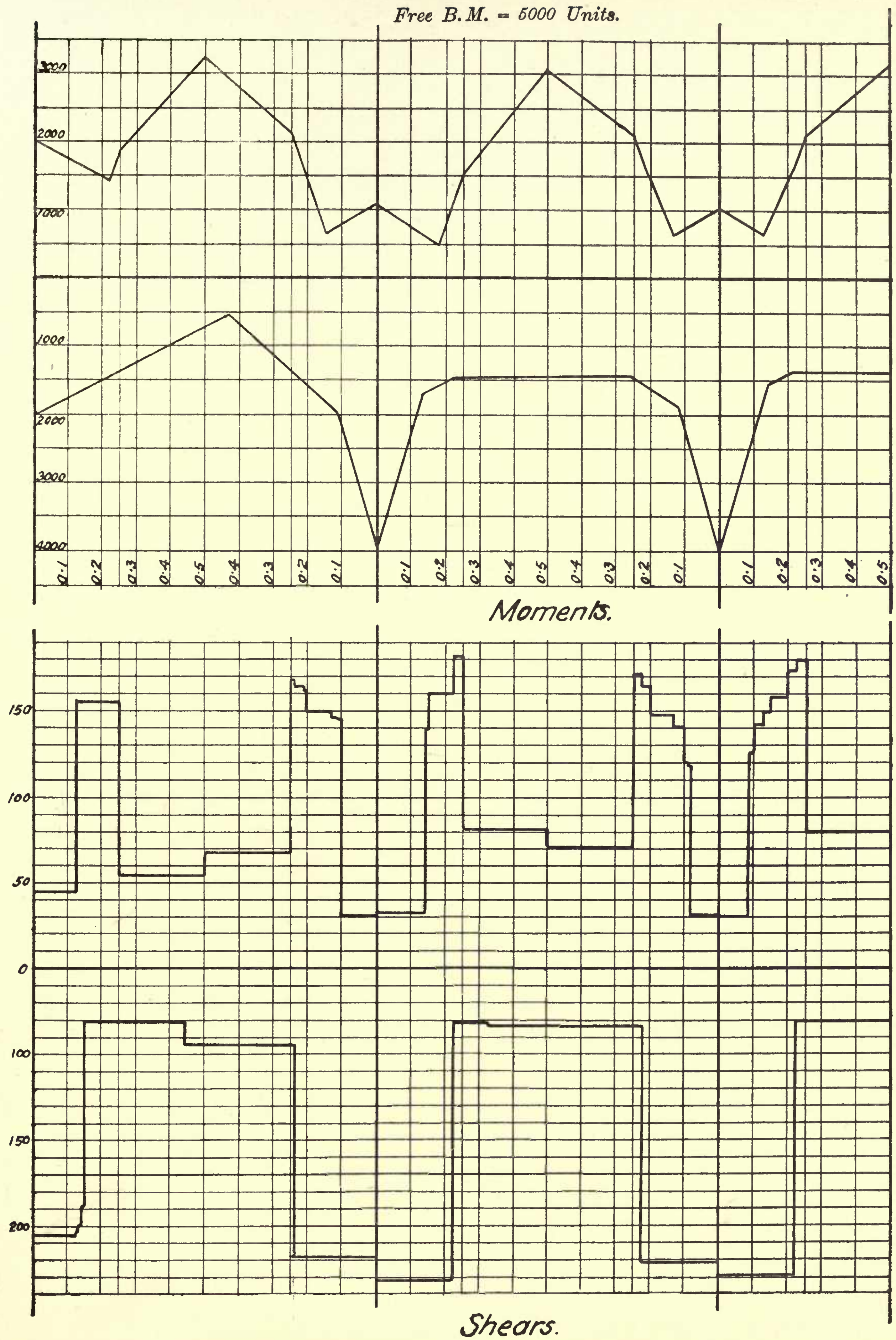


Moments.



Shears.

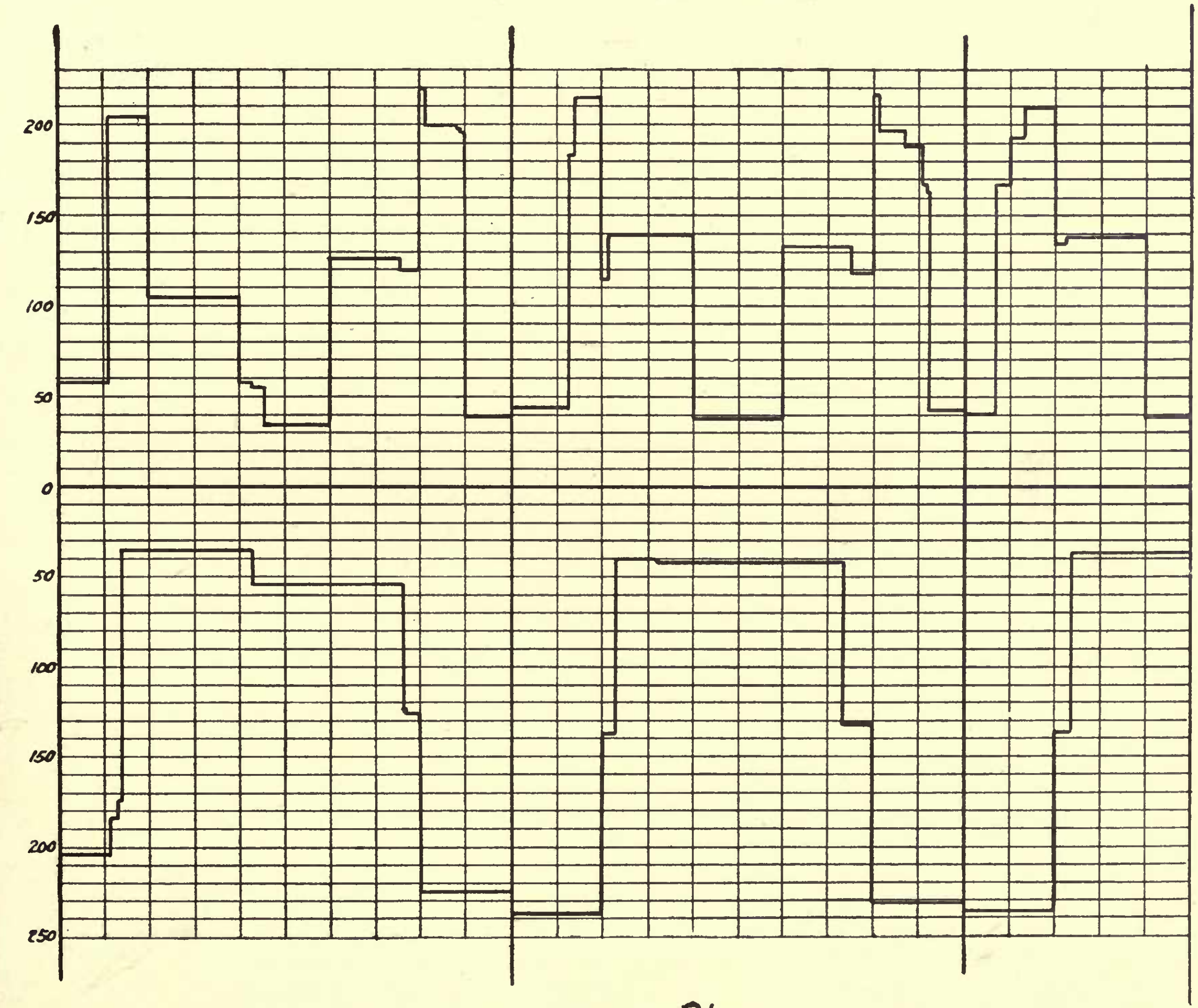
DIAGRAM No. 67



z

DIAGRAM Nº 68.

Moments.



Shears.

Diagram No. 69.

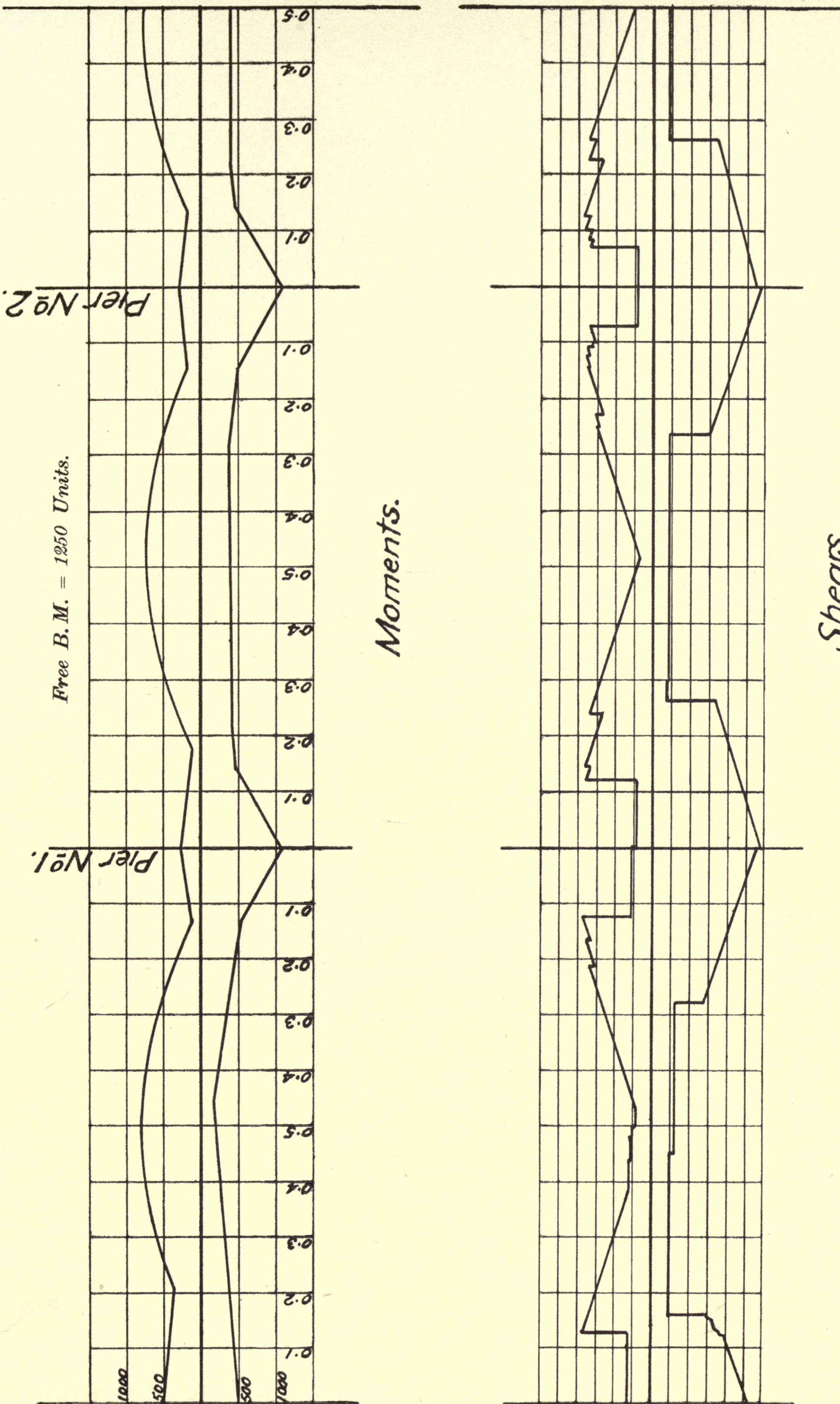


Fig. 5.

Point in Span.	− B. M. Dead.	− B. M. Live.	− B. M. Total.	+ B. M. Dead.	+ B. M. Live.	+ B. M. Total.	− Shear Dead.	− Shear Live.	− Shear Total.	+ Shear Dead.	+ Shear Live.	+ Shear Total.
Outer Support	0	0	0	0	0	0	0	4,520	0	15,30	31,100	46,400
0·1	—	—	—	—	—	—	—	—	—	—	—	—
0·15	—	—	—	—	—	—	—	—	—	—	—	—
0·2	0	267,000	0	768,000	1,872.000	2,640,000	0	4,520	0	7,700	31 100	38,800
0·25	—	—	—	—	—	—	0	4,520	0	5,800	13,200	19,000
0·3	—	—	—	—	—	—	—	—	—	—	—	—
·35	—	—	—	—	—	—	—	—	—	—	—	—
0·4	0	534,000	0	975,000	2,680,000	3,655,000	0	4,520	4,520	0	13,200	13,200
0·45	—	—	—	—	—	—	0	4,520	1,520	3,000	10,400	13,400
0·5	—	—	—	—	—	—	—	—	—	—	—	—
0·45	—	—	—	—	—	—	0	4,520	0	7,200	10,400	17,600
0·4	0	806,000	180,000	626,000	2,285,000	2,911,000	0	4,520	0	9,200	28,300	37,500
0·35	—	—	—	—	—	—	—	—	—	—	—	—
0·3	—	—	—	—	—	—	0	4,520	0	13,300	28,300	41,600
0·25	—	—	—	—	—	—	15,300	28,700	43,600	15,300	28,300	43,600
0·2	252,000	1,075,000	1,327,000	0	1,040,000	788,000	17,200	46,200	63,400	0	41,000	33,800
0·15	—	—	—	—	—	—	—	—	—	—	—	—
0·1	—	—	—	—	—	—	21,000	46,200	67,200	0	41,000	20,000
Pier 1	1,260,000	3,095,000	4,355,000	0	371,000	0	24,700	46,200	70,900		1,240	0
Pier 1	1,260,000	3,095,000	4,355,000	0	371,000	0	20,700	44,000	64,700	0	5,640	0
0·1	—	—	—	—	—	—	16,700	44.000	60,700	0	36.800	20,100
0·15	—	—	—	—	—	—	—	—	—	—	—	—
0·2	420,000	1,145,000	1,565,000	0	862,000	442,000	12,700	44,000	56,700	0	36,800	24,100
0·25	—	—	—	—	—	—	10,800	26,500	37,300	0	26,200	15,400
0·3	—	—	—	—	—	—	8,800	20,600	29,400	8,800	26.200	17,400
0·35	—	—	—	—	—	—	—	—	—	—	—	—
0·4	0	1,102,000	825,000	277,000	2,000,000	2,277,000	0	8,800	3,800	5,000	26,200	31,200
0·4	—	—	—	—	—	—	—	—	—	—	—	—
0·5	—	—	—	—	—	—	—	—	—	—	—	—
0·45	—	—	—	—	—	—	—	—	—	—	—	—
0·4	0	1,064,000	707,000	357,000	2,070,000	2,427,000	0	8,800	5,900	2,900	25,200	28,100
0·35	—	—	—	—	—	—	0	8,800	3,800	5,000	25,200	30,200
0·3	—	—	—	—	—	—	0	16,500	9,700	6,800	25,200	32,000
0·25	—	—	—	—	—	—	8,800	25,200	34,000	8,800	25,200	34,000
0·2	189,000	1,130,000	1,319,000	0	1,065,000	876,000	10,900	43.100	4,000	0	35,800	24,900
0·15	—	—	—	—	—	—	—	—	—	—	—	—
0·1	—	—	—	—	—	—	15,100	43,100	58,200	0	35,800	20,700
Pier 2	942,000	2,890,000	3,832,000	0	832,000	0	19,300	43,100	62,400	0	27,900	0
Pier 2	942,000	2,890,000	3,832 000	0	832,000	0	20,000	43,100	63,100	0	30 400	0
0·1	—	—	—	—	—	—	16,000	43,100	59,100	0	37,500	21,500
0·15	—	—	—	—	—	—	14,000	43,100	57,100	0	41,300	27,300
0·2	0	1,018,000	1,018,000	0	1,150,000	1,150,000	12,000	43,100	55,100	0	41,300	29,300
0·25	—	—	—	—	—	—	11,000	25,200	36,200	11,000	25,200	36,200
0·3	—	—	—	—	—	-	0	7,800	0	8,000	25,200	33,200
0·35	—	—	—	—	—	—	—	—	—	—	—	—
0·4	0	1,018,000	525,000	493,000	2,100,000	2,593,000	0	7,800	3,800	4,000	25,200	29,200
0·45	—	—	—	—	—	—	—	—	—	—	—	—
0·5	0	1,018,000	468,000	555,000	1,976,000	2,531,000	0	7,800	7,800	0	7,300	7,300

NOTE.—The necessary corrections for increased reactions have been purposely omitted in the above. See Chap V.

CHAPTER VIII.

SUGGESTED FORM FOR CALCULATION SHEETS OF BEAMS.

A CONVENIENT method of collecting the various determining figures for the computation of the section of a beam is as shewn in Fig. 5, and this table will be found to go very conveniently into an ordinary foolscap book, sideways, down two pages.

The figures shewn in this Fig. 5 are those assumed in Chapter XIV.

By this means all the required information is capable of being inspected at one time; and although it may appear at first sight to be rather unnecessary elaboration, it will be found to be ultimately of very real advantage from the point of view of the facility with which the figures can be checked, and in the case of a member of a structure which is repeated many times, a little extra care is amply repaid, if even only a small economy can be effected in the member.

In addition, it serves to shew the effect of the dead load as compared with the live load; and shews to what extent it offsets possible change of sign of moment due to live load. The maximum moments, both positive and negative, at any point, are shewn; and this information is necessary, besides serving to shew how far it is economical to use steel in compression for resisting any given moment. The steel stressed in tension for a given positive moment is stressed in compression under a negative moment at the same point in the span. The difference in bending moment between any two points in the span is obtained by inspection, and the stirrups can be readily proportioned to resist shear as previously explained; the vertical shears being also given at any point.

The amounts of the moments and shears gives in Fig. 5. are for 5 spans, with free ends at the outer supports. Where a small amount only of fixedness of ends is allowed, the necessary negative moments and shears can be put into the above figures without much error.

TABLES GIVING BENDING MOMENTS AND SHEARS, FIXED ENDS, 40 per cent.

It will be observed that the foregoing diagrams 40–54 and 55–69 are arranged so as to give the required moments and shears at points equidistant along the span, the beam being divided into ten parts, and that in the tables 6–8 and the following tables 9–11 each of these lengths is again subdivided, making a total of 20 parts in each beam.

TABLES OF REACTIONS, FIXED ENDS, 40 per cent.

The tables, numbers 12–14, give the reactions from any series of beams, and the reader is referred to Chapter V. for further information on this question.

Table

Two Spans.

The Bending Moments are expressed in terms of percentages of the Maximum Free

Cases A and F, and in percentages of one point

Point in Span.	Dead Load.				1 Point Load.				2 Point Loads.			
	B. Ms.		Shears.		B. Ms.		Shears.		B. Ms.		Shears.	
Outer Support	0	40	0	0·445	40	40	0·194	0·505	40	40	0·3	0·962
0·1	0	8	0·305	0·351	32·3	20·8	0·194	0·505	31	13·3	0·3	0·962
0·15	5	0	0·262	0·305	28·4	12·9	0·194	0·505	26·5	0	0·962	0·962
0·2	16	0	0·218	0	24·5	4	0·505	0·505	22	0	0·962	0
0·25	25	0	0·174	0	20·6	0	0·505	0	33·1	0	0·962	0
0·3	32	0	0·131	0	23	0	0·505	0	47·3	0	0·962	0
0·33	—	—	—	—	—	—	—	—	56·8	0	0·962	0
0·35	37	0	0·087	0	32·6	0	0·505	0	56·6	0	0·132	0
0·4	40	0	0·04	0	42·2	0	0·505	0	56·1	0	0·132	0
0·45	41	0	0	0	51·7	0	0·505	0	55·6	0	0·132	0·3
0·5	40	0	0·04	0	61·3	0	0·505	0·194	55·1	5	0·132	0·3
0·45	37	0	0·087	0	53·6	3·7	0·542	0·194	54·6	9·5	0·132	0·3
0·4	32	0	0·131	0	46	7·5	0·542	0·194	54·1	14	0·132	0·3
0·35	25	0	0·174	0	38·3	11·2	0·542	0·194	53·6	18·5	0·132	0·3
0·33	—	—	—	—	—	—	—	—	53·4	20·3	1·132	0·3
0·3	16	0	0·218	0	30·6	15	0·542	0·542	44·4	23	1·132	0·3
0·25	5	0	0·262	0·305	23	18·8	0·542	0·542	29·6	27·5	1·132	1·132
0·2	0	8	0·305	0·355	15·3	22·5	0·495	0·542	14·8	32	1·132	1·132
0·15	0	23	0	0·405	7·7	27·8	0	0·542	0	36·5	1·038	1·132
0·1	0	40	0	0·455	0	33	0	0·542	0	46·7	0	1·132
Pier No. 1	0	80	0	0·555	0	55	0	0·542	0	80	0	1·132
	+	—	+	—	+	—	+	—	+	—	+	—
	Case A.				Case B.				Case C.			

No. 9.
Fixed Ends.

Bending Moment ; and the Shears in percentages of the total load on one span in load in the remaining Cases B, C, D, and E.

3 Point Loads.				***4 Point Loads.***				***Distributed Load.***			
B. Ms.		***Shears.***		***B. Ms.***		***Shears.***		***B. Ms.***		***Shears.***	
40	40	0·434	1·48	40	40	0·541	1·955	40	40	0·112	0·48
31·3	13·3	1·48	1·48	31·2	10	2·055	1·955	31·2	8	0·112	0·406
27	0	1·48	1·48	26·8	0	2·055	1·955	26·8	0	0·34	0·34
22·6	0	1·48	0	24·7	0	2·055	0	22·4	0	0·287	0
33·3	0	1·48	0	32·5	0	1·055	0	32·5	0	0·234	0
38	0	0·48	0	40·3	0	1·055	0	41	0	0·182	0
—	—	—	—	—	—	—	—	—	—	—	—
42·7	0	0·48	0	48·1	0	1·055	0	47·5	0	0·130	0
47·3	0	0·48	0	56	0	1·055	0	52	0	0·077	0
52	0	0·48	0·434	55·5	0	0·242	0·541	54·5	0	0·025	0·112
56·6	3·5	0.656	0·434	55	4·9	0·242	0·541	55	5	0·072	0·112
51·2	7·8	0·656	0·434	54·5	9·5	0·242	0·541	53·5	9·5	0·118	0·112
45·9	12·1	0·656	0·434	54	14	1·242	0·541	50	14	0·165	0·112
40·5	16·5	0·656	0·434	45·1	18·5	1·242	0·541	44·5	18·5	0·211	0·112
—	—	—	—	—	—	—	—	—	—	—	—
35·2	20·8	0·656	0·434	36·3	23	1·242	0·541	37	23	0·258	0·112
29·8	25·2	1·656	1·656	27·5	27·5	1·242	1·242	27·5	27·5	0·305	0·305
17	29·5	1·656	1·656	18·7	32	2·055	2·242	16	32	0·26	0·355
0	33·9	1·52	1·656	0	36·5	2·055	2·242	2·5	36·5	0·30	0·405
0	41	0	1·656	0	43·1	0	2·242	0	41	0·34	0·455
0	73·7	0	1·656	0	80	0	2·242	0	80	0	0·555
+	—	+	—	+	—	+	—	+	—	+	—
Case D.				***Case E.***				***Case F.***			

Table
Three Spans.

The Bending Moments are expressed in terms of percentages of the Maximum Free
Cases A and F ; and in percentages of one point

Point in Span.	Dead Loads.				1 Point Load.				2 Point Loads.			
	B.Ms.		Shears.		B.Ms.		Shears.		B.Ms.		Shears.	
Outer Support	0	40	0	0·47	40	40	0·195	0·516	40	40	0·293	1·044
0·1	0	7	0	0·39	36·5	21·8	0·195	0·516	32	33·8	1·044	1·044
0·15	6·2	0	0·30	0·34	31·3	12·9	0·516	0·516	28	30·8	1·044	1·044
0·2	17·6	0	0·25	0·30	22	3·6	0·516	0·516	24	27·7	1·044	0·205
0·25	27	0	0·2	0	25·5	0	0·516	0·455	35·9	24·6	1·044	0·205
0·3	34·3	0	0·15	0	23	0	0·516	0·145	52·2	21·5	1·044	0·205
0·33	—	—	—	—	—	—	—	—	63	19·7	1·044	0·205
0·35	39·7	0	0·1	0	34·5	0	0·516	0·145	63·1	18·5	0·293	0·205
0·4	43·1	0	0·05	0	46	0	0·516	0·145	63·5	15·3	0·293	0·205
0·45	44·5	0	0	0	57·5	0	0·516	0·145	63·8	12·3	0·293	0·293
0·5	43·4	0	0·043	0	69	0	0·545	0·195	64·2	9·2	0·13	0·293
0·45	41·3	0	0·085	0	59·8	3	0·545	0·195	64·5	9·7	0·13	0·293
0·4	36·7	0	0·128	0	50·6	69	0·545	0·195	64·9	13·1	0·13	0·293
0·35	30·1	0	0·17	0	41·5	10·8	0·545	0·195	65·2	17·5	0·13	0·293
0·33	—	—	—	—	—	—	—	—	65·4	19·2	1·13	0·293
0·3	21·5	0	0·213	0	32·2	14·7	0·545	0·545	57	21·8	1·13	0·293
0·25	10·9	0	0·256	0	23	18·6	0·545	0·545	42·7	26·2	1·13	1·13
0·2	0	2	0·30	0·30	13·8	22·6	0·484	0·545	28·8	30·6	1·1	1·13
0·15	0	16·4	0	0·36	9·3	26·5	0·484	0·545	14·2	34·9	1·1	1·13
0·1	0	33	0	0·41	12·2	33·9	0·455	0·545	15·3	45·2	0·956	1·13
Pier No. 1	0	72·2	0	0·53	18	54·5	0·014	0·545	21·4	80	0·205	1·13
Pier No. 1	0	72·2	0	0·50	18	54·5	0·012	0·558	21·4	80	0·223	1·13
0·1	0	36·2	0	0·396	13	32·7	0·012	0·558	14·7	45·2	0·223	1·13
0·15	0	21·2	0	0·344	10·5	21·8	0·442	0·558	11·4	35·3	0·87	1·13
0·2	0	8·2	0	0·292	8	22	0·5	0·558	12	32	1·13	1·13
0·25	2·8	0	0·24	0·24	15·5	22	0·558	0·558	27	32	1·13	1·13
0·3	11·8	0	0·192	0	20·7	22	0·558	0·5	42	32	1·13	0·233
0·33	—	—	—	—	—	—	—	—	52	32	1·13	0·233
0·35	18·8	0	0·144	0	31	22	0·558	0·125	52	32	1·13	0·233
0·4	23·8	0	0·096	0	41·3	22	0·558	0·125	52	32	1·13	0·233
0·45	26·8	0	0·048	0	51·7	22	0·558	0.125	52	32	1·13	0·233
0·5	27·8	0	0	0	62	22	0·558	0·125	52	32	1·13	0·233
	+	—	+	—	+	—	+	—	+	—	+	—
	Case A.				Case B.				Case C.			

No. 10.
Fixed Ends.

Bending Moment ; and the Shears in percentages of the total load on one Span in load in the remaining Cases B, C, D, and E.

3 Point Loads.				*4 Point Loads.*				*Distributed Load.*			
B.Ms.		*Shears.*		*B.Ms.*		*Shears.*		*B.Ms*		*Shears.*	
40	40	0·48	1·558	40	40	0·528	2·027	40	40	0·36	0·53
32·1	33·9	1·558	1·558	32	33·9	2·027	2·027	32	33·9	0·296	0·435
28·2	30·9	1·558	1·588	24	30·8	2·027	2·027	30	30·8	0·264	0·387
24·2	27·9	1·558	0·303	28·3	27·7	2·027	0·368	27·9	27·7	0·232	0·34
37·6	24·8	1·558	0·303	37	24·6	1·027	0·368	37	24·6	0·2	0·077
43·1	21·8	0·558	0·303	45·7	21·5	1·027	0·368	46·4	21·6	0·168	0·077
—	—	—	—	—	—	—	—	—	—	—	—
48·7	18·8	0·558	0·303	54·4	18·5	1·027	0·368	53·8	18·5	0·136	0·077
54·2	15·8	0·558	0·48	63·2	15·4	1·027	0·368	59·2	15·4	0·104	0·077
59·7	12·7	0·558	0·48	63·6	12·4	0·528	0·528	62·6	12·4	0·072	0·077
65·3	9·7	0·665	0·48	64	9·3	0·48	0·528	64	9·3	0·04	0·077
60·8	6·7	0·665	0·48	64·4	8·2	0·264	0·528	63·4	9·1	0·09	0·11
56·3	14·7	0·665	0·48	64·8	12·6	1·264	0·528	60·8	13·4	0·14	0·11
51·9	18·5	0·665	0·48	56·8	17·1	1·264	0·528	56·2	17·7	0·19	0·11
—	—	—	—	—	—	—	—	—	—	—	—
47·4	22·4	0·665	0·48	48·9	21·5	1·264	0·528	49·6	22	0·25	0·27
42·9	26·3	1·665	1·665	41	25·9	1·264	1·264	41	26·4	0·25	0·316
28·6	30·1	1·665	1·665	33·1	30·3	2·165	2·264	30·4	30·7	0·32	0·362
14·3	33·9	1·52	1·665	17	34·8	2·165	2·264	17·8	35	0·32	0·408
14·5	41·2	1·52	1·665	15·2	44·4	2·01	2·264	15·4	40	0·36	0·454
20·5	74·2	0·303	1·665	21·3	80	0·368	2·264	21·5	80	0·077	0·545
20·5	74·2	0·313	1·70	21·3	80	0·398	2·25	21·5	80	0·073	0·55
14·3	41·2	1·3	1·70	14·7	44·4	1·75	2·25	15·4	40	0·073	0·438
11·2	32·6	1·5	1·70	11·4	38·3	2·0	2·25	12·3	37	0·34	0·382
14·8	29·5	1·7	1·70	18·7	35	2·0	2·25	16	32	0·30	0·326
29·5	29·5	1·7	1·70	27	32	1·25	1·25	27	32	0·27	0·27
34·5	29·5	0·7	0·313	35·4	32	1·25	0·398	36	32	0·226	0·24
—	—	—	—	—	—	—	—	—	32	—	0·073
39·5	29·5	0·7	0·313	43·7	32	1·25	0·398	43	32	0·182	0·073
44·5	29·5	0·7	0·313	52	32	1·25	0·398	48	32	0·138	0·073
49·5	29·5	0·7	0·313	52	32	0·25	0·398	51	32	0·094	0·073
54·5	29·5	0·7	0·313	52	32	0·25	0·398	52	32	0·05	0·073
+	−	+	−	+	−	+	−	+	−	+	−
Case D.				*Case E.*				*Case F.*			

Table No. 11.

Five Spans. Fixed Ends.

The Bending Moments are expressed in terms of percentages of the Maximum Free Bending Moment; and the Shears in percentages of the total load on one span in Cases A and F, and in percentages of one point load in the remaining Cases B, C, D, and E.

Point in Span.	Dead Load.				1 Point Load.				2 Point Loads.				3 Point Loads.				4 Point Loads.				Distributed Load.			
	B. Ms.		Shears.		B. Ms.		Shears.		B. Ms.		Shears.		B. Ms.		Shears.		B. Ms.		Shears.		B. Ms.		Shears.	
Outer Support	0	40	0	0·457	40	40	0·205	0·547	40	40	0·31	1·023	40	40	0·45	1·55	40	40	0·557	2·038	40	40	0·116	0·515
0·1	0	7·4	0	0·357	34·7	35	0·205	0·547	34·7	34·8	1·023	1·023	34·7	34·9	1·55	1·55	34·7	34·7	2·038	2·038	34·7	34·9	0·385	0·3
0·15	5·9	0	0·307	0·307	32·1	32·5	0·205	0·547	32·1	32·3	1·023	1·023	32	32·3	1·55	1·55	32	32·2	2·038	2·038	32·0	32·3	0·35	0·3
0·2	17·2	0	0·256	0	29·5	30	0·547	0·547	29·4	29·7	1·023	0·207	29·4	29·8	1·55	0·306	29·4	29·4	2·038	0·374	29·5	29·7	0·3	0·107
0·25	24·5	0	0·204	0	26·8	27·5	0·547	0·45	36·5	27·1	1·023	0·207	37·5	27·2	1·55	0·306	36·8	27·8	1·038	0·374	36·9	27·1	0·25	0·107
0·3	33·8	0	0·153	0	25·5	25	0·547	0·152	52	24·6	1·023	0·207	43	24·6	0·55	0·306	45·5	24·1	1·038	0·557	46·2	24·6	0·2	0·107
0·33	—	—	—	—	—	—	—	—	62·5	23·1	1·023	0·207	—	—	—	—	—	—	—	—	—	—	—	—
0·35	39·1	0	0·102	0	36·4	22·5	0·547	0·152	62·9	22	0·31	0·207	48·5	22	0·55	0·306	54·3	21·5	1·038	0·557	53·6	22	0·15	0·107
0·4	42·4	0	0·051	0	47·3	20	0·547	0·152	63	19·4	0·31	0·31	54·1	19·4	0·55	0·306	63	18·8	1·038	0·557	59	19·4	0·116	0·107
0·45	43·7	0	0	0	58·2	17·5	0·547	0·152	63·4	16·8	0·31	0·31	59·6	16·9	0·55	0·45	63·3	16·2	0·557	0·557	62·4	16·8	0·116	0·112
0·5	43	0	0·044	0	69·2	15	0·564	0·205	63·7	14·3	0·274	0·31	65·1	14·3	0·683	0·45	63·7	13·6	0·54	0·557	63·7	14·3	0·116	0·112
0·45	40·3	0	0·088	0	60·2	17·5	0·564	0·205	64·1	11·7	0·25	0·31	60·6	11·7	0·683	0·45	64·1	11·6	0·54	0·557	63·1	11·7	0·1	0·112
0·4	35·6	0	0·132	0	51·2	10	0·564	0·205	64·5	15·7	0·178	0·31	56·1	14	0·683	0·45	64·5	16·1	1·262	0·557	60·5	15·5	0·15	0·112
0·35	28·9	0	0·176	0	42·2	13·4	0·564	0·205	64·9	20·3	0·178	0.31	51·6	18·5	0·683	0·45	56·6	20·6	1·262	0·557	55·9	20·2	0·2	0·112
0·33	—	—	—	—	—	—	—	—	65	22·2	1·155	0·31	—	—	—	—	—	—	—	—	—	—	—	—
0·3	20·2	0	0·22	0	33·1	17·5	0·564	0·564	51	25	1·155	0·31	47·1	23	0·683	0·45	48·6	24·7	1·262	0·557	49·2	24·8	0·29	0·29
0·25	9·5	0	0·264	0	24·1	21·6	0·564	0·564	37·1	29·6	1·155	1·155	42·6	27·5	1·683	1·683	40·6	29·7	1·262	1·262	40·6	29·5	0·31	0·33
0·2	0	3·2	0·307	0·307	15·1	25·7	0·533	0·564	23·2	34·2	1·155	1·155	28·4	32	1·683	1·683	32·6	34·2	2·2	2·262	30	34·1	0·35	0·38
0·15	0	17·9	0	0·365	10	29·8	0·49	0·564	12·7	38·8	1·03	1·155	14·2	36·5	1·5	1·683	12·7	38·75	2·0	2·262	17·4	38·8	0·37	0·42
0·1	0	34·6	0	0·423	13·5	38·7	0·453	0·564	15·8	51	1·03	1·155	15·2	49·6	1·5	1·683	9·1	50·9	2·0	2·262	15	50·8	0·385	0·47
Pier No. 1	0	74	0	0·542	19·1	60·8	0·152	0·564	22	84·8	0·207	1·155	21·3	78·8	0·306	1·683	22	84·8	0·372	2·262	22	84·8	0·107	0·555

Pier No. 1	0	74	0	0·507	19·1	60·8	0·139	0·603	22	84·8	0·233	1·255	21·3	78·8	0·332	1·813	22	84·8	0·42	2·383	22	84·8	0·088	0·57
0·1	0	37·1	0	0·38	13·5	36·5	0·139	0·603	15·1	48·4	0·9	1·255	14·6	49·6	1·4	1·813	15	44·5	2·135	2·383	13·4	50·8	0·36	0·48
0·15	0	21·6	0	0·316	10	24·3	0·139	0·603	9·3	35·3	1·255	1·255	11·3	31·8	1·6	1·813	11·5	35·9	2·135	2·383	9·6	35·3	0·36	0·44
0·2	0	8·2	0·252	0·252	7·9	21·9	0·55	0·603	11	32	1·255	1·255	15·3	29·5	1·813	1·813	18·7	32·4	2·135	2·383	15·9	32	0.31	0·39
0·25	3·2	0	0·21	0	11·3	22·0	0·603	0·603	27·5	31·9	1.255	1·255	30·6	29·5	1·813	1·813	28·2	31·7	1·383	1·383	28·1	31·8	0·337	0·34
0·3	12·7	0	0·168	0	22·7	22·1	0·603	0·12	44	31·8	1·255	0·233	36·7	29·4	0·813	0·332	37·7	31·6	1·383	0·42	38·2	31·7	0·287	0·088
0·33	—	—	—	—	—	—	—	—	55·	31·7	1·255	0·233	—	—	—	—	—	—	—	—	—	—	—	—
0·35	20·1	0	0·126	0	33·9	22·1	0·603	0·139	55·4	31·6	0·255	0·233	42·9	29·3	0·813	0·332	47·2	31·5	1·383	0·42	46·4	31·5	0·237	0·088
0·4	25·6	0	0·084	0	45·3	22·2	0·603	0·139	56·6	31·5	0·255	0·233	49	29·2	0·813	0·332	56·7	31·4	1·383	0·42	52·6	31·4	0·187	0·088
0·45	29·1	0	0·042	0	56·7	22·2	0·603	0·139	57·7	31·4	0·255	0·233	55·2	29·1	0·813	0·332	58·2	31·3	0·383	0·42	56·7	31·2	0·137	0·088
0·5	30·5	0	0	0	68·2	22·3	0·603	0·139	58·9	31·3	0·255	0·233	61·3	29	0·715	0·332	59·8	31·2	0·383	0·42	59	31·1	0·087	0·088
0·45	29·9	0	0·042	0	57·8	22·4	0·577	0·139	60·1	31·2	0·255	0·233	57·4	28·9	0·715	0·332	61·3	31·1	0·383	0·42	59·1	31	0·1	0·088
0·4	27·4	0	0·084	0	47·4	22·4	0·577	0·139	61·2	31·0	0·255	0·233	53·5	28·8	0·715	0·332	62·9	30·9	1·31	0·42	57·2	30·8	0·15	0·088
0·35	22·8	0	0·126	0	37	22·5	0·577	0·139	62·4	30·9	0·255	0·233	49·7	28·7	0·715	0·332	55·4	30·8	1·31	0·42	53·4	30·7	0·2	0·088
0·33	—	—	—	—	—	—	—	—	62·8	30·8	1·165	0·233	—	—	—	—	—	—	—	—	—	—	—	—
0·3	16·3	0	0·168	0	26·6	22·5	0·577	0·577	53	30·7	1·165	0·233	45·8	28·6	0·715	0·332	48	30·6	1·31	0·42	47·5	30·5	0·25	0·088
0·25	7·7	0	0·21	0	16·2	22·7	0·577	0·577	38·6	30·6	1·165	1·165	41·9	28·5	1·715	1·715	40·5	30·5	1·31	1·31	39·7	30·5	0·30	0·30
0·2	0	2	0·252	0·252	6·2	25·5	0·56	0·577	24·2	34·1	1·165	1·165	32·6	31·9	1·715	1·715	33·1	31·2	2·147	2·31	29·9	34·1	0·30	0·36
0·15	0	15·3	0	0·312	8·6	28·7	0·485	0·577	12	37·5	0·997	1·165	22·4	35·2	1·48	1·715	19·3	34·4	2·147	2·31	18·0	37·6	0·365	0·41
0·1	0	29	0	0·372	11	40·2	0·485	0·577	15·4	48·6	0·997	1·165	14·3	50·2	1·48	1·715	15·4	47·5	1·945	2·31	16·2	52·2	0·365	0·47
Pier No. 2	0	65	0	0·492	15·7	63·2	0.12	0·577	22·2	85·0	0·745	1·165	20·6	79·8	1·203	1·715	22·2	85·4	1·865	2·31	22·2	85·2	0·337	0·585
Pier No. 2	0	65	0	0·50	15·7	63·2	0·12	0·613	22·2	85·0	0·77	1·217	20·6	79·8	1·259	1·80	22·2	85·4	1·628	2·372	22·2	85·2	0·37	0·555
0·1	0	29	0	0·384	11	38·8	0.48	0·613	15·4	47·2	0·97	1·217	14·5	50·2	1·5	1·80	15·7	44·7	2·09	2·372	15·7	51·2	0·36	0·47
0·15	0	14	0	0·327	8·6	26·5	0·50	0·613	12·4	33·4	1·055	1·217	13·4	35·5	1·577	1·80	19·7	33·4	2·09	2·372	18·5	33·5	0.35	0·43
0·2	0	1	0·27	0·27	14	23·9	0·50	0·613	24·2	30·2	1·217	1·217	33·6	28	1·8	1·80	33·7	30·2	2·09	2·372	30·6	30·2	0·35	0·39
0·25	10	0	0·225	0	23·3	21·6	0·613	0·613	37·4	29·6	1·217	1·217	42·6	27·5	1·8	1·80	41	29·5	1·372	1·372	40·6	29·5	0·35	0·35
0·3	19	0	0·18	0	32·8	21·6	0·613	0·12	50·7	29·6	1·217	0·218	46·8	27·5	0·8	0·305	48·3	29·5	1·372	0·392	48·7	29·5	0·3	0·082
0·33	—	—	—	—	—	—	—	—	64	29·6	1·217	0·218	—	—	—	—	—	—	—	—	—	—	—	—
0·35	26	0	0·135	0	42·2	21·6	0·613	0·12	63·7	29·6	0·217	0·218	50·9	27·5	0·8	0·305	55·6	29·5	1·372	0·392	54·8	29·5	0·25	0·082
0·4	31	0	0·09	0	51·5	21·6	0·613	0·12	62·8	29·6	0·217	0·218	55·1	27·5	0·8	0·305	62·9	29·5	1·372	0·392	58·9	29·5	0·2	0·082
0·45	34	0	0·045	0	60·9	21·6	0·613	0·12	61·9	29·6	0·217	0·218	59·2	27·5	0·8	0·305	62	29·5	0·372	0·392	61·0	29·5	0·15	0·082
0·5	35	0	0	0	70·2	21·6	0·613	0·12	61	29·6	0·217	0·218	63·4	27·5	0·8	0·305	61·1	29·5	0·372	0·392	61·1	29·5	0·1	0·082
	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—	+	—
	Case A.				*Case B.*				*Case C.*				*Case D.*				*Case E.*				*Case F.*			

MULTIPLES FOR REACTIONS—ENDS FIXED 40%

Reactions expressed in terms of percentages of full load on one span in Cases A and F, and in percentages of one point load in remaining Cases B, C, D, and E.

Table No. 12. Two Spans.

CASE.	LOAD.	OUTER SUPPORT.	PIER 1.
A	Dead load	0·305	1·11
B	One point load	0·505	1·08
C	Two ,, ,,	0·97	2·26
D	Three ,, ,,	1·48	2·31
E	Four ,, ,,	1·95	2·48
F	Distributed ,,	0·48	1·11

Table No. 13. Three Spans.

CASE.	LOAD.	OUTER SUPPORT.	PIER 1.
A	Dead load	0·47	1·03
B	One point load	0·545	1·103
C	Two ,, ,,	1·044	2·26
D	Three ,, ,,	1·558	3·365
E	Four ,, ,,	2·027	2·514
F	Distributed ,,	0·49	1·1

Table No. 14. Five Spans.

CASE.	LOAD.	OUTER SUPPORT.	PIER 1.	PIER 2.
A	Dead load	0·457	1·05	0·992
B	One point load	0·547	1·174	1·19
C	Two ,, ,,	1·023	2·41	2·382
D	Three ,, ,,	1·55	3·49	3·515
E	Four ,, ,,	2·038	4·64	4·68
F	Distributed ,,	0·515	1·135	1·14

CHAPTER IX.

HAUNCHES TO BEAMS.

A CAREFUL study of the tables is recommended, for from them much useful information may be gathered It will be observed that the maximum positive and negative moments along the span are very dissimilar under the live loads, and immediately adjacent to the intermediate supports the negative moments are largely in excess.

Provided there is no architectural objection to the introduction of large haunches at the

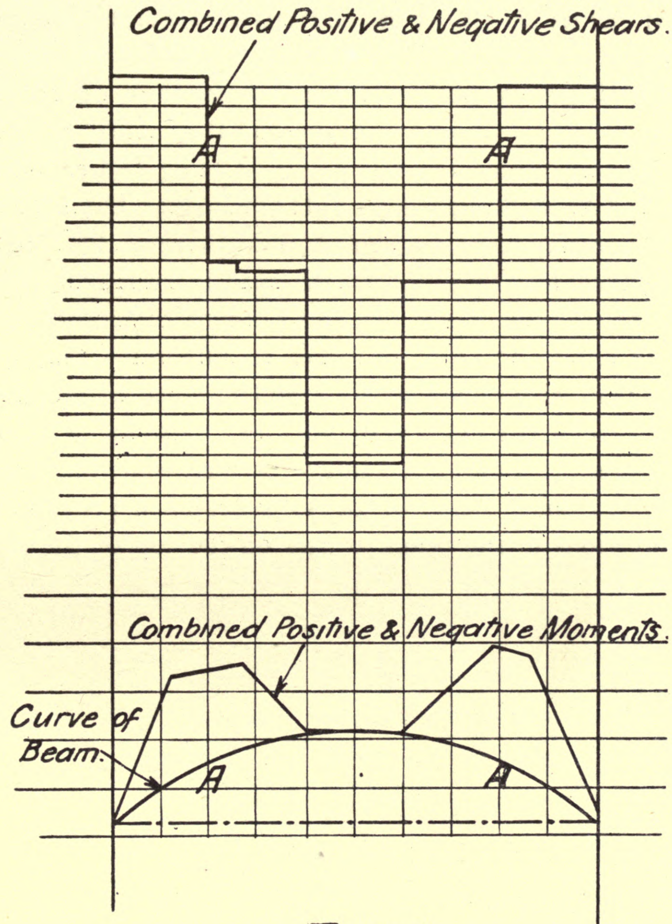


Fig 6.

piers, the insertion of haunches properly considered as part of the increased depth of the beam at the supports forms by far the most economical and satisfactory method of dealing with the excess of negative moment previously mentioned ; as the portion of the beam between the haunches may now be designed to suit the lower limits of positive and negative bending moment as given in the tables.

Where the angular haunches are considered objectionable for the reason aforesaid, or other reason, the best solution of large span rectangular beams with heavy loads is undoubtedly to make the soffit of the beams curved. The particular curve best suited to any particular case may be found by combining both positive and negative moments due to dead and live loads combined in one diagram on one side of a base line, and joining up or including the limiting points so obtained ; Fig. 6 shews one inner span of a series of five, with 4 point loads in which, however, the dead load has been omitted. By joining the points of limiting maximum B.M. we obtain a curve which gives the necessary covering depth of beam at any point in the span ; and if these depths are expressed in terms of the maximum by drawing a horizontal line between the piers and through the points of maximum moment in each case, the desired curve of beam is obtained by first determining the maximum section at the pier to suit the B.M. and shear at that point. Where it is permissible to use **T** beam action in the beams, this is of course not true ; and it is then necessary to calculate the required depth of beam at several points in the span, and strike a curve passing as far as possible through the points so found.

By plotting the shear diagrams, both positive and negative on the same base line, we get the limiting shears to be carried at any section ; and these must be checked on the curve of beam found for the bending moments ; the points A being the critical points.

It will also be observed from the tables and diagrams that the distances from the centres of piers to which the haunching must be carried out to meet the foregoing suggestions is in every case something much less than 0·1 of the span in rectangular beams, and less than 0·1 in **T** beams ; a requirement which can be met without difficulty.

It is very general to allow moderate haunches to important beams, without, however, taking full advantage of them, as outlined above ; the additional cost of making such allowed haunches into fully efficient ones is almost negligible, and by doing it a considerable economy can in many cases be effected. Another advantage of increasing the depth of the beams at the supports is that, if this is done, there will be no difficulty in providing room for the steel coming from opposite directions in the beam to pass on the top of the beam at the supports.

CHAPTER X.

EFFECT OF SUBSIDENCE OF SUPPORTS, PREVIOUSLY ASSUMED RIGID.

IF the supports of a series of continuous beams, originally set to the contour of the beams in their normal unstrained condition, should subside unequally relatively to one another, then the bending moments and shears undergo considerable change, according to the amount of subsidence.

In the case of main beams running into columns it can reasonably be assumed that the intermediate columns are rigid (the actual relative diminution in length of any two columns due to compression being negligible), as great precautions are generally taken to ensure moderate ground pressures in the case of column footings.

The case is, however, very different with secondary beams supported on main beams, or slabs supported on secondary beams.

The main beams supporting the secondary beams have only a certain limited degree of stiffness, and if we assume, for the sake of example, that the main beams will deflect $\frac{1}{640}$ of

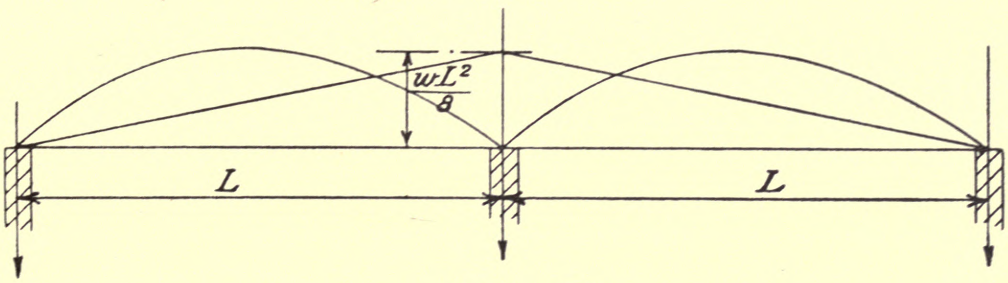


Fig 7.

their span, we shall see that this deflection or subsidence, in the case of unequal loading on the secondary beams, will have an effect upon the resulting moments and shears in those beams.

Considering a series of two 16′ 0″ spans of secondary beams, if we allow the centre support to sink 0·3″ in relation to the outer adjacent supports, it will be equivalent to offsetting the amounts of the theoretical moments for two continuous spans by an amount due to considering a span of secondary beam equal to two actual spans, or 32 feet, with a deflection of 0·3″.

This will have the effect of reducing the tension in the upper surface of both the secondary beams over the main beam and of increasing the tension on the under side of both beams in the centre portions between the main beams.

An example will be worked out to shew what is meant. Let us assume for simplicity that a steel beam 16″ × 6″ × 62 lbs. per ft. is continuous over two spans of 16′ 0″ each, and is loaded with a similar load to that of the secondary beams in Chapter XIV.

If we assume rigid supports, the bending moment diagram is as shewn in Fig. 7.

If now the centre support sinks away entirely, the bending moment diagram is as shewn in Fig. 8, or if, on the other hand, the centre pier elongates or rises considerably, or the two

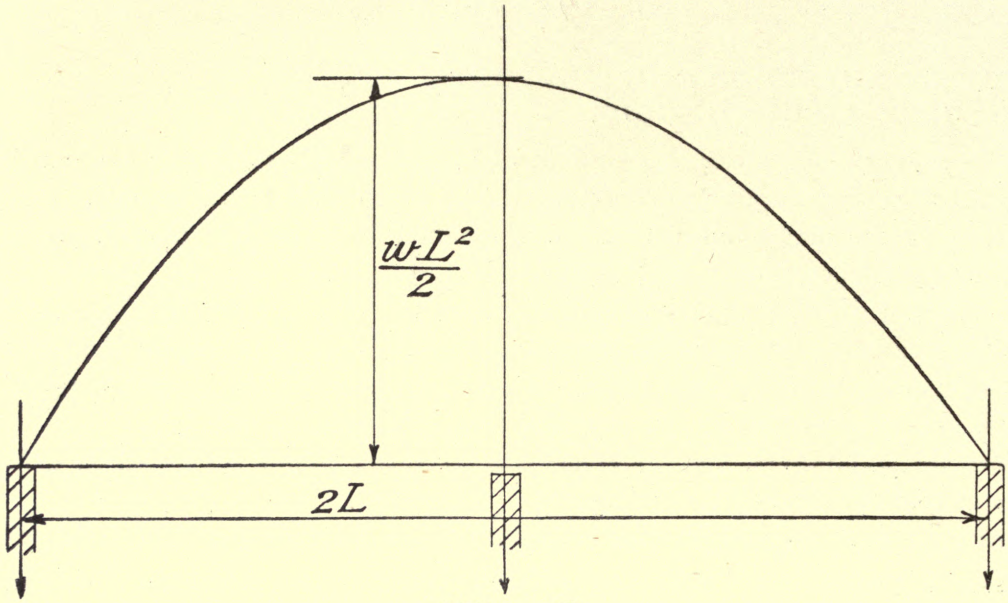


Fig 8.

end supports sink entirely, then the bending moment diagram would be as shewn in Fig. 9.

In the case of the secondary beams under consideration, it is obvious that these extreme

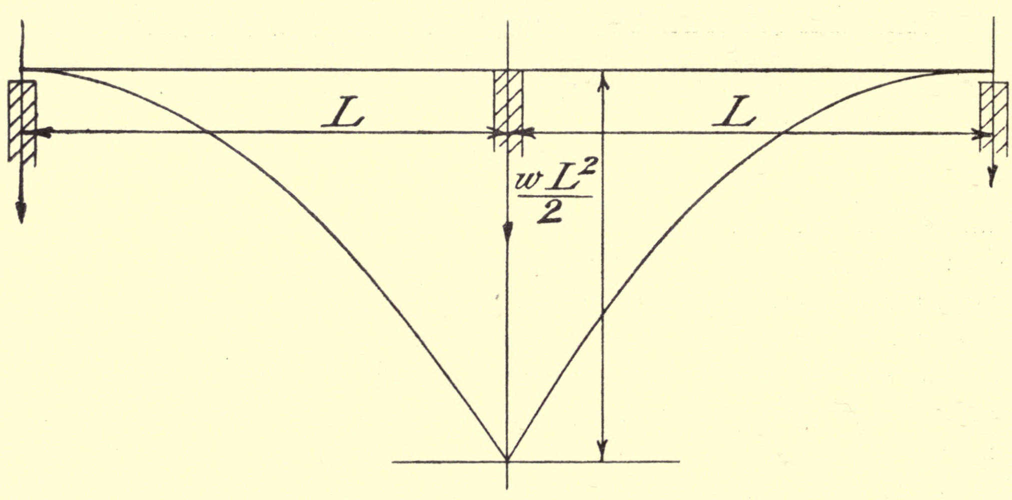


Fig 9.

conditions do not apply, since our subsidence is small. It is necessary, therefore, to determine what effect the given subsidence will have upon the bending moment diagram, Fig. 7.

The bending moments are measured by the elastic deflection of the beam, and if we wish

to find the change in bending moment effected by a subsidence of the centre support, we must consider the elastic deflection.

The span = 16′ 0″ in case of Figs. 7 or 9, and 32 ft. in case of Fig. 8.

The moment of inertia of the beam is 725·7 inch units.

The deflection of main beam is 0·3″.

In Fig. 6 the bending moment at pier is

$$\frac{5 \times 16^2 \times \overset{\text{lbs.}}{324} \times 12}{8} = \underline{622{,}080 \text{ inch-pounds}}$$

and the top flange stress equals

$$\frac{622{,}080 \times 8}{725\cdot7 \times 2240} = \underline{3\cdot06 \text{ tons per sq. in. tension.}}$$

If we now assume that the centre pier has sunk as in Fig. 7, and the two end spans are acting as one of length 32′ 0″, then the bending moment equals

$$\frac{32^2 \times 5 \times 324 \times 12}{8} = 2{,}488{,}320 \text{ inch-pounds}$$

and the flange stress equals

$$\frac{2{,}488{,}320 \times 8}{725\cdot7 \times 2240} = \underline{12\cdot24} \quad \text{(compression in tons per sq. in. in top flange)}$$

and the corresponding deflection will be

$$\frac{5 \times 12\cdot24 \times 32^2 \times 12 \times 12}{24 \times 13{,}000 \times 16} = 1\cdot8''*$$

therefore the centre support must sink 1·8″ before this derived stress of 12·24 tons per sq. in. can be reached.

If now, on the contrary, the end supports have sunk relatively to the centre support, then the stress again will be 12·24 tons per sq. in. tension in the top flange, and the corresponding deflection will be

$$\frac{12\cdot24 \times 16^2 \times 12 \times 12}{2 \times 13{,}000 \times 16} = 1\cdot08''\dagger$$

therefore the end supports must sink 1·08″ relatively to the centre support before the derived stress of 12·24 tons per sq. in. can be reached.

The change of stress from + 12·24 tons per sq. in. to − 12·24 tons per sq. in. requires a total difference or range of subsidence of 1·8″ + 1·08″, which equal 2·88″.

These changes take place in the same proportion for any subsidence between these limits, and therefore if we are dealing with a deflection of the centre support of 0·3″ the tensile stress in the top flange over the pier will now become $12\cdot24 - \left(\frac{1\cdot5}{2\cdot88} \times 24\cdot48\right) = -0\cdot51$ tons per sq. in. as compared with 3·06 tons per sq. in. in Fig. 6.

* This result may also be obtained from the formula $\frac{5 W l^3}{384 E I} = 1\cdot8''$

† This result may also be obtained from the formula $\frac{W l^3}{8 E I} = 1\cdot08''$

With the deflections usually obtained in continuous reinforced concrete beams ranging from $\frac{1}{2000}$ to $\frac{1}{5000}$ of the span, it will be found that the deflection of one beam relative to the next is small, being only the difference between the deflection of the main beam where the load leaves off and the deflection in the next main beam covered by the load; and this difference is only about one-half of the actual maximum deflection.

In the foregoing numerical example a steel beam is assumed for simplicity, but a reinforced concrete beam may be taken as a parallel case.

It will be observed that for medium or long spans of secondary beams, with deflections of main beams of less than $\frac{1}{3000}$ of their span, the increase in stress is not serious, amounting to under 5 per cent. in normal cases, and may be ignored except in the case of very short spans of secondary beams or slabs, where the deflection is likely to be greater in proportion.*

But if the deflection is considerable, as much as $\frac{1}{1000}$ of the span of the main beam, then the negative bending at the pier may be reduced very considerably, and the positive moments in the centre portion increased to a corresponding degree. This points to the necessity for great stiffness in the main beams, and the author is of opinion that the permissible deflection therein should not exceed $\frac{1}{3000}$ of their span, when the effect of this deflection upon the secondary beams may be ignored.†

Coming now to the question of the slab, we see that not only is there the deflection of the secondary beam to consider, but the main beam also deflects. The result is therefore worse in the case of slabs than in secondary beams. The deflection too in the case of secondary beams is greater in proportion than the main beams and seriously affects the assumption of rigid supports. Whereas the effect on secondary beams is, generally speaking, below 5 per cent, the effect on slabs is more likely to be anything from 15 to 20 per cent., and allowance should be made in designing slabs to meet this effect.

In the opinion of the author the usual assumption of $\frac{w\,L^2}{12}$ for the moment at the support and at the centre of the span of slabs is therefore sufficiently near for practical requirements where the supports are liable to deflection, as is the case with secondary beams. Where the supports are more or less rigid this moment is not sufficient, and in the case of slabs running over walls of short height, for example, the full theoretical moment should be allowed.

* Considering 2 spans, as per Fig. 7, a deflection of $\frac{1}{5000}$ of the span would give about 10% decrease in negative moment, and the increase in positive moment would be about 5%. When there are several spans this increase will be slightly smaller, and will in fact be found to be less than 5% for deflections of main beams of $\frac{1}{3000}$ of their span. It should be remembered that the main beams are generally of smaller span than the secondary beams, though this is not always the case.

† The deflections given are considered as being caused by the total designed load, and may be exceeded if under a test load greater than this amount.

CHAPTER XI.

ASSUMPTION OF POWER TO MEET AN ARRANGEMENT OF BENDING MOMENTS BY AN INFINITE NUMBER OF ARRANGEMENTS OF RESISTING MOMENTS.

ASSUMING that the moment of inertia is constant and taking the critical moments due to any arrangement of loading, it has been contended that if the centre portion of the beam is made stronger than is required to meet the theoretical bending moments there, then the resisting moments at the piers need not be made strong enough to carry the theoretical bending moments at that point, but may be reduced by an amount dependent upon the excess of strength in the centre position.

In other words, if we increase the length of the virtual detached span in the centre, we can decrease the length of the cantilevers at the sides; or, conversely, if we make the resisting moments at the piers stronger than is necessary to meet the theoretical moments there, we can reduce the strength of the resisting moments in the virtual detached span in the centre, by an amount dependent upon the excess of strength in the cantilever portions; or, in other words, we decrease the length of the virtual detached span in the centre.

The following Figs. 10 and 11 will show more clearly what is meant.

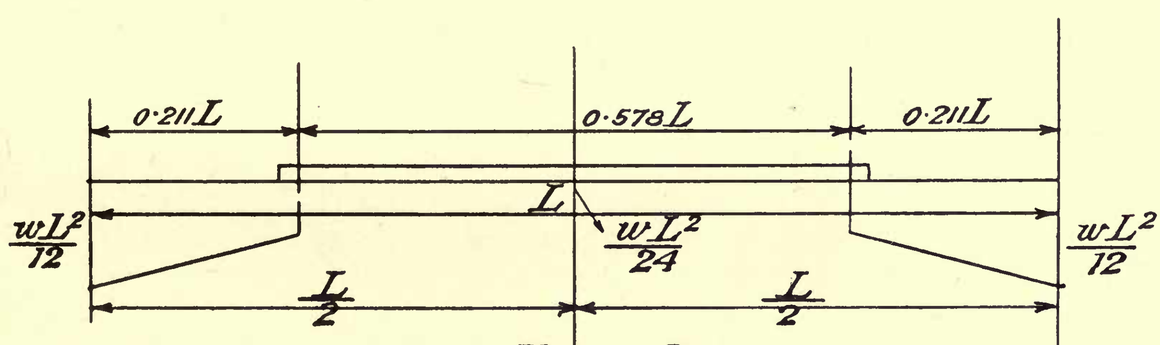


Fig 10.

In Fig. 10, on the assumption of constant moment of inertia, the virtual span is 0·578 L in length, if all spans are loaded with distributed load and each cantilever 0·211 L in length, in order to give the theoretical moments $\frac{w\,L^2}{24}$ and $\frac{w\,L^2}{12}$ respectively.

If, however, as in Fig. 11, we transpose these moments, the corresponding virtual spans become 0·8164 L and 0·0918 L at each pier.

In other words, the bending moments can be met in an infinite number of ways between the limits of a single span of length L, with a bending moment $\frac{w\,L^2}{8}$ (if we assume the

span cut through at the pier) and two cantilevers with spans $\frac{L}{2}$ each and bending moments $\frac{w L^2}{8}$ each (if we assume the beams cut through at the centre of the span); and each of these would carry the loads.

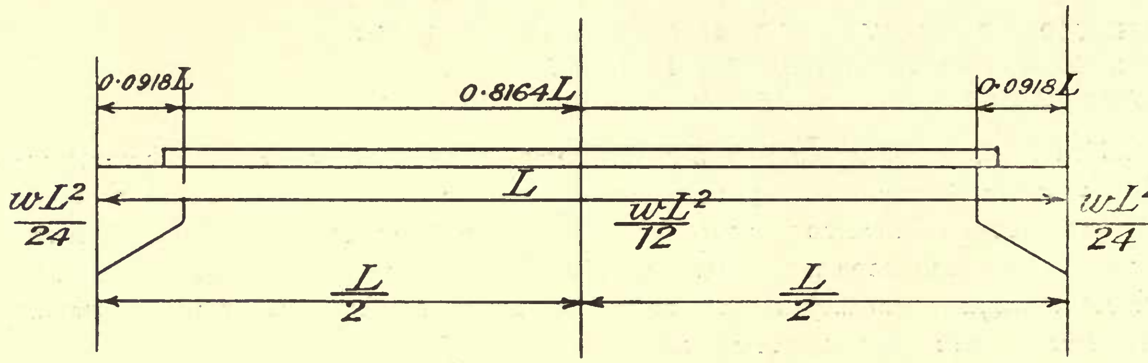


Fig 11.

In considering the theory, however, it must be borne in mind that if we assume spans equal to L and cut at the piers, and provide a resisting moment accordingly, there will be some slight moment at the pier and cracks will develop there until the beam is released and free to act as a simple beam, which it would then do, carrying the full bending moment of $\frac{w L^2}{8}$ in the centre and o at the piers.

It must also be borne in mind that the cantilevers have to be balanced, which (so as not to rely upon any load on the adjoining spans) would require a negative resisting moment in the adjoining spans, these beams suffering negative flexure and bending moment.

If we assume, as in Fig 10, that at the points of contraflexure we were to arrange the beam so that it was equivalent to being hinged there, the detached portion taking a moment of $\frac{w L^2}{24}$, and each cantilever a moment of $\frac{w L^2}{12}$, we should have complied with all requirements for that particular loading.

If we deal with Fig. 11 in the same way, by altering the points of contraflexure to new positions nearer the supports, we should have again met all requirements for that particular loading, and it has been put forward that if any continuous beam spanning between two supports is capable of being split up into two cantilevers and a detached span, the resisting moments of which are capable of carrying the bending moments equivalent to the actual theoretical moments, then we may consider the beam sufficiently strong, apart from the fact that the theoretical moments on the assumption of constant moment of inertia are not met.

The following Fig. 12 shews the bending moment diagram for one detached span of a series, loaded with 4 point loads, erected upon a base line x x.

If we now raise this base line so as to occupy any position above this line such as A A, B B, C C, either horizontal or inclined, and the resisting moments of the beam are arranged to equal the resulting moments and shears given by the diagrams so formed, then

it is suggested that the beam will carry the required loads, always assuming that the necessary resisting moments or the equivalents are provided in the adjoining spans.

Tests have been made in support of the foregoing theory, but the author does not in any way consider that they have shewn it to be true, and is inclined to think that the long arm of coincidence has had a good deal to do with the results, and the deflection of supporting beams still more.

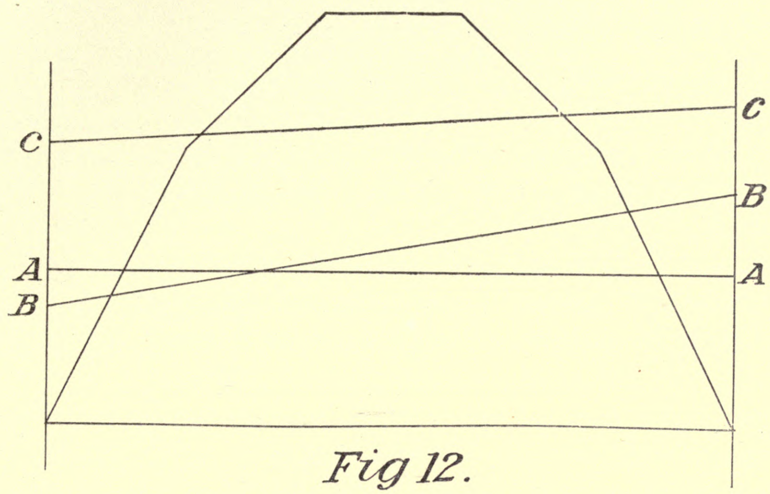


Fig 12.

The difficulty of dealing with the dead load and live load at one operation is very great, and for that reason they have been dealt with as two separate and distinct loads in the diagrams and tables in this book. By this means accurate results can be obtained, and it can be readily seen to what extent the dead load offsets reversal of stress by making calculations in the form of Fig. 5.

Speaking generally, in the case of all main beams and secondary beams of the requisite stiffness and having moderate deflections of under $\frac{1}{3000}$ of the span, and in cases of slabs supported on comparatively rigid supports, the author is of opinion that the bending moments and shears given in the diagrams and tables should be adopted, as, not only can greater economy be effected by so doing, but greater safety can be secured by the avoidance of as many unjustifiable or doubtful assumptions as possible.

CHAPTER XII.

TEST LOADS ON COMPLETED STRUCTURES.

THE whole question of testing parts of a complete structure is very unsatisfactory. Firstly, it is obvious that the most important members in a structure are the columns (if any), and with, perhaps, the exception of top lengths of columns supporting roofs, and water loads carried on columns, as in the case of water towers, &c., it is practically impossible to test them at all.

To test a slab is a fairly simple matter, and generally is done easily enough, though it should be observed that it is necessary to test a sequence of slabs with various arrangements of loading before the test can be considered a satisfactory proof of its strength under all conditions, the maximum stresses being obtained by loading alternate bays with live load. A careful study of diagrams 1–39 will shew what is necessary in this respect.

Coming now to the question of secondary beams, it is necessary to load two complete adjacent bays of slab before we get our required load on them, and all the possible arrangements of loading must be applied as in the case of the slab before the test can be considered as in any way conclusive.

Next, considering main beams, it will be seen of course that we have to employ a very large loaded area, amounting to a full panel of secondary beams on either side of the main beams under consideration. Then again, as in the case of the secondary beams and slabs, it is necessary to try all the various arrangements of loading on a sequence of spans before the test can be considered as conclusive. It is not suggested in the foregoing remarks that the tests usually adopted do not entirely in some cases, and in some few particulars in all cases, give useful results ; but it is believed that the results obtained are inconclusive.

Without this variation in loading it is quite impossible to speak definitely upon the strength of the work, and as such elaborate tests are hardly ever carried out on any large scale, on the score of the great expense entailed (except, perhaps, in the case of moving loads), the only conclusion to be drawn is that if the money spent in testing completed work by loading as at present carried out were devoted to extra supervision during construction, and extra tests of concrete were made while the work was going forward at the site, the results would stand a good chance of being better for the same expenditure.

In certain important cases, however, these rather expensive tests mentioned might be advantageously undertaken.

CHAPTER XIII.

COMPARISON OF TABLES AND DIAGRAMS WITH TESTS ON COMPLETED STRUCTURES.

DURING 1911 and part of 1912 certain tests were made in America upon completed buildings, with a view to determining the actual stresses in various members of the structures.

These tests were carried out by the Committee of Reinforced Concrete of the National Association of Cement Users of America, and, as far as they go, give some interesting results.

They are not in themselves complete, and it would be of inestimable value to the profession generally if further tests were carried out (which it is to be hoped will be done) upon similar lines, but with a complete arrangement of all the possible critical cases of loading.

The tests measured extensions or compressions in the steel and concrete under increasing loads on a number of consecutive panels of floors, and also the resulting deflections at various points in the beams. Special care was taken in the measuring instruments to make the readings as accurate as possible, proper allowance being made for temperature.

The floors, which were similar to Case c in the tables with main beams carrying two secondary beams, were designed with certain assumed bending moments of $\frac{w L^2}{12}$ at the supports and at the centre of the spans, and the resulting stresses are compared with these assumed bending moments.

The results as they stand are not conclusive, but bearing in mind the particulars of the design, and the methods of observation which have a direct influence upon the results, they may be taken as giving a general indication of the results to be expected. Much credit is due to those responsible for the tests, and when the final report is issued it will be of considerable interest. The tests apply chiefly to the secondary beams, the test on the main beam being incomplete.

The beams were loaded with distributed load over three adjacent spans, the loads being applied first on the centre span, and secondly on all three spans. The beams were of constant depth, and were so arranged as to have the amount of steel in the bottom at the centre of the span twice the amount of the steel over the supports, the span of secondary beam being 20′ 0″, and main beam 15′ 0″.

The following Tables 15 and 16 shew the figures derived from the tests, and the theoretical figures, in which the ends of the beams are assumed first free and then fixed 40 per cent., derived from Tables Nos. 8 and 11.

It will be seen that there is considerable divergence between the results, which, however, can to some extent, if not entirely, be explained.

Table 15 deals with an intermediate beam running into a main beam; and Table 16 deals with a beam running into a column; and the effect of the extra rigidity of the supports

is clearly shown by the greater moment at the end of the beam in Table 16, and conversely the effect of the subsidence of the supports is shewn by the decrease of moments at the ends of the beam in Table 15.

The effect of the subsidence of the supports has been touched upon, and it is obvious that this subsidence of the main beams is in a great degree responsible for the difference, as the positive moments are found to be greater than the negative moments in the tests in Table 15, which is the opposite to the theoretical calculation; but in accordance with the assumption upon which the beams were designed.

Table No. 15.

Beams designed for B.M. $= \frac{w\,L^2}{12}$ at support and at centre of span $= 0{\cdot}083\,w\,L^2$.

INTERMEDIATE SECONDARY BEAM.

B.Ms. expressed in terms of $w\,L^2$.

	Test No. 1. Derived B.Ms.		Test No. 2. Derived B.Ms.		Theoretical calculation. B.Ms.	
	From Steel Stress.	From Concrete Stress.	From Steel Stress.	From Concrete Stress.	Free ends.	Fixed ends.
End of beam ……	0·037	0·088	0·03	0·07	0·094	0·0898
Centre of beam …	0·06	—	0·05	0·077	0·0684	0·0614

NOTE.—The tests Nos. 1 and 2 were on different buildings, having similar arrangements of beams.

Table No. 16.

SECONDARY BEAM RUNNING INTO COLUMNS.

B.Ms. expressed in terms of $w\,L^2$.

	Test No. 1. Derived B.Ms.		Test No. 2. Derived B.Ms.		Theoretical calculations. B.Ms.	
	From Steel Stress.	From Concrete Stress.	From Steel Stress.	From Concrete Stress.	Free ends.	Fixed ends.
End of beam ……	—	—	—	0·064	0·094	0·0898
Centre of beam …	—	—	0·05	0·054	0·0684	0·0614

In considering the effect of subsidence of supports in Chapter X., it is pointed out that, with moderate deflections, an increase of about 5% in the positive moment is all that need be anticipated in the case of secondary beams; but in comparing the positive and negative moments as we are doing at present this result will be exceeded, as not only are we increasing the positive moment but we are decreasing the negative moment also.

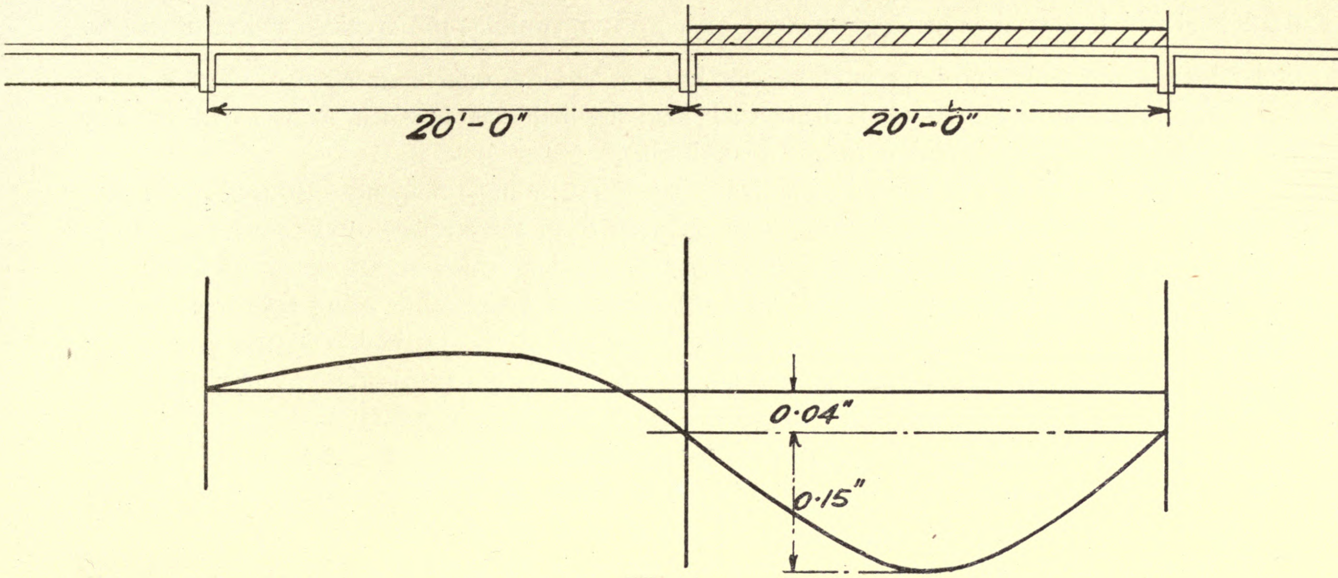


Fig 13.

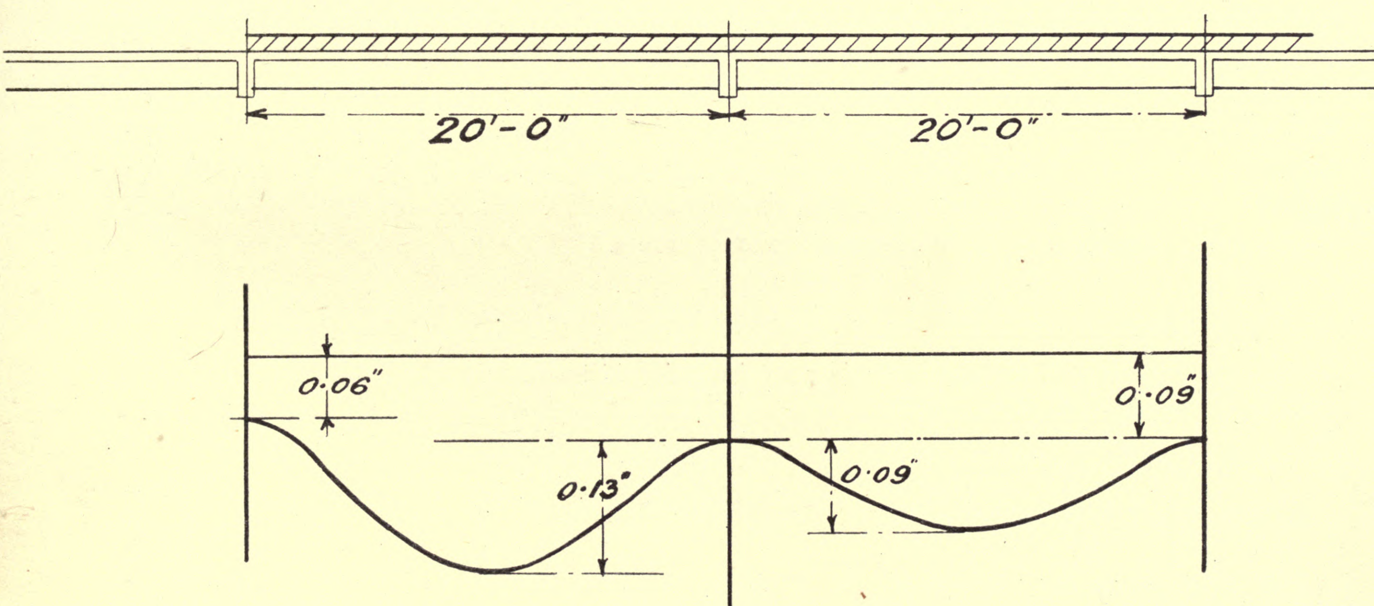


Fig 14.

Possible further causes of difference are the increase of $-$ B.M. at the supports for the dead load which was taken as $\frac{w\,L^2}{12}$ instead of a greater amount, the arching action of beams under load, about which nothing is at present known, and the tensile strength of the concrete. Care was taken in the tests that the actual load applied was of such form as to prevent any possibility of arching action in the load itself.

One peculiarity of the tests is the fact that the bending moments deduced from the stresses in the concrete are greater than those deduced from the stresses in the steel, suggesting arching action in and tensile strength of the concrete, and pointing to an excess of steel.

The beams were calculated as **T** beams, the width of flange table being taken as the full width between the centres of beams. This was proved to be justifiable in the case under test, where the width of slab between beam faces was about thirteen times its thickness, the measured stress in the concrete being constant throughout the width.*

In comparing the tests and the calculations, the dead load of 75 lbs. per sq. ft. has been worked out and combined with the live load of 400 lbs. per sq. ft. to arrive at the results and the bending moments are expressed in terms of $w\,L^2$.

The approximate deflection of the secondary beams as tested is shewn in Figs. 13 and 14, and from them will be seen the deflection of the main beams also.

This measured deflection, it will be noticed, is given as $= 0$ at the main beams next to the loaded spans; the reason the foregoing theoretical calculations are taken as for five spans being that it would suit the case of three bays loaded; in addition to which the difficulty of measuring the deflection of main beam $= 0$ may have rendered this a trifle inaccurate; or possibly if the column was stiff in proportion to the beam, the column may have suffered bending moment.

In calculating the bending moments from the observed stresses, the ratio of the moduli of elasticity of steel to concrete was taken as 10 in test No. 1, and 13 in test No. 2, the stresses being worked out on the assumption of a straight line stress strain curve, which is probably accurate for the lower stresses, though a parabolic curve probably more nearly expresses the relationship in the higher limits of stress.†

* The recent Report of the R.I.B.A. gives 15 times the thickness.

† The information as to these tested results is to be found in *Engineering News*, April 18th, 1912.

CHAPTER XIV.

EXAMPLES OF APPLICATION OF TABLES AND DIAGRAMS.

LET us assume that it is required to find the designing moments and shears on the centre one of a series of five main beams with free ends, each 25′ 0″ span, loaded with a live load of 2 cwt. per sq. ft., the beams being 16′ 0″ apart and carrying secondary beams 5′ 0″ apart, equally spaced, giving 4 point loads on each main beam ; there being many spans of secondary beams.

Live Load.—Taking the moments and shears due to the live load first, we have each point load = 2 cwt. × 112 lbs. × 16 ft. × 5 ft. = 17,920 lbs.

The simple free bending moment for a detached span at point 0·4 from pier is 3,225,600 inch-pounds.

Referring to Table No. 8 and Case E, we find that the maximum positive bending moment at the centre is 65 % of the simple free bending moment which equals $\frac{65}{100}$ × 3,225,600 = roughly 2,110,000 inch-pounds, and so on in like manner for the remaining positive and negative moments throughout the span.

Each point load on the beam is 17,920 lbs. ; and referring to the same Table 8, Case E, we find the percentage multiplier for the maximum negative shear = 2·41, and giving 2·41 × 17,920 lbs. = 43,200 lbs. roughly, and so on in like manner for the remaining negative and positive shears throughout the span.

Dead Load.—Having obtained the live load moments and shears, the next step is to roughly approximate the sections of the beams and slabs, so as to obtain the approximate weight of the equally distributed dead load, which in the present case we will assume as 100 lbs. per sq. ft.

The simple free bending moment at the centre for a detached span, considering the total load as being equally distributed, is 1,500,000 inch-pounds; and referring to Table 8, Case A, for dead load on all spans, we find the maximum positive moment at the centre is 37 % of the maximum simple free bending moment at the centre, which gives $\frac{37}{100}$ × 1,500,000 = 555,000 inch-pounds, and so on in like manner for the remaining positive and negative moments throughout the span.

The total load on the beam is 25 ft. × 16 ft. × 100 lbs. = 40,000 lbs., and if we multiply this amount by the percentages given for the shears, we find the maximum negative shear equals 40,000 lbs. × 0·5 = 20,000 lbs., and so on in like manner throughout the span for the remaining negative and positive shears.

Designing Moments.—The next step is to add up the live and dead negative moments and shears and the live and dead positive moments and shears as shewn in Fig. 5, and if on designing the section of beam it is found to be appreciably larger than that assumed

for the purpose of calculating the dead loads, the necessary adjustment must be made ; a little practice, however, will allow of sufficiently accurate approximations being made at the first attempt.

Referring to table in Fig. 5, it will be noticed that the algebraic sum of positive and negative moments and shears is made, and the correct designing moments and shears are derived, the correct amount of reversal of stress being thus obtained.

In the foregoing figures no allowance has been made for increased reactions on the main beams from the secondary beams, and Chapter V. should be referred to for the necessary particulars. If it is decided to consider an increase of 10 °/。 on the main beams due to live load and 13 °/。 due to dead load, as per Example 5, Chapter V., the revised figures would become

+ B.M. live load	=	2,110,000	× 1·1	=	2,321,000 inch-lbs.
— Shear ,, ,,	=	43,200	× 1·1	=	47,520 lbs.
+ B.M. dead load	=	555,000	× 1·13	=	627,150 inch-lbs.
— Shear ,, ,,	=	20,000	× 1·13	=	22,600 lbs.

and so on for the remaining moments and shears throughout the span.

This is on the assumption, as previously explained, that the beams are stiff in relation to the columns, and that the columns are acting as props only, and not carrying any bending moment.

APPENDIX I.

THEOREM OF THREE MOMENTS: GENERAL CASE.

We have, on page 4, the following statements :—

$$M_a = M_1\left(\frac{A - x}{A}\right) + M_2\frac{x}{A} - m_a$$

$$\text{and also } M_b = M_3\frac{z}{B} + M_2\left(\frac{B - z}{B}\right) - m_b$$

Also work done in bending beam across the two spans

$$= \frac{1}{2E}\left\{\int_0^A \frac{(M_a)^2}{I}(dx) + \int_0^B \frac{(M_b)^2}{I}(dz)\right\}$$

Here $(M_a)^2$ is a function of M_a, and M_a is a function of M_2; therefore

$$\frac{d\,(M_a)^2}{dM} = \frac{d\,(M_a)^2}{d\,(M_a)} \times \frac{d\,M_a}{d\,(M_2)} = 2\,M_a \times \frac{x}{A}$$

$$= 2\left\{\frac{M_1\,(A - z)}{A} + \frac{M_2\,x}{A} - m_a\right\}\frac{x}{A}$$

since M_2 is the only variable in the expression for M_a.

$$\text{Similarly } \frac{d\,(M_b)^2}{d\,M_2} = 2\left\{M_3\frac{z}{B} + M_2\left(\frac{B - z}{B}\right) - m_b\right\}\frac{B - x}{B}$$

$$\text{therefore } \frac{du}{dM_2} = \frac{1}{2E}\left\{\int_0^A \frac{2}{I}\left[\frac{M_1\,(A - x)}{A} + \frac{M_2\,x}{A} - m_a\right]\frac{x}{A}(dx)\right.$$

$$\left. + \int_0^B \frac{2}{I}\left[\frac{M_3\,z}{B} + M_2\frac{B - z}{B} - m_b\right]\frac{B - z}{B}(dz)\right\}$$

$$= \frac{1}{E}\left\{\int_0^A\left[M_1\frac{A - x}{A} + M_2\frac{x}{A} - m_a\right]\frac{x}{A}\frac{(dx)}{I} + \int_0^B\left[M_3\frac{z}{B} + M_2\frac{B - z}{B} - m^b\right]\frac{B - z}{B}\frac{(dz)}{I}\right\}$$

Equating to zero for a minimum and multiplying by E.

$$\int_0^A\left\{\left(M^1\frac{A - x}{A} + M_2\frac{x}{A} - m_a\right)\frac{x}{A}\frac{(dx)}{I}\right\}$$

$$= -\int_0^B\left\{(M_2)\frac{B - z}{B} + M_3\frac{z}{B} - m_b\right)\frac{B - z}{B}\frac{(dz)}{I}\right\}$$

which is the general expression from which any case can be solved.

Assuming I to be constant and multiplying out

$$\left\{\int_0^A \left(\frac{M_1(A\,x - x^2)}{A^2} + \frac{M_2\,x^2}{A^2}\right)(d\,x) - \int_0^A \frac{m_a\,x}{A}(d\,x)\right\}$$

$$= -\left\{\int_0^B \frac{M_2(B^2 - 2\,B\,z + z^2)}{B^2} + \frac{M_3(B\,z - z^2)}{B^2}(d\,z) - \int_0^B m_b\frac{B - z}{B}(d\,z)\right\}$$

Integrating for M_1 and M_2, but not m_a and m_b.

$$M_1\left(\frac{\frac{A\,x^2}{2} - \frac{x^3}{3}}{A^2}\right) + \frac{M_2\,x^3}{3\,A^2} - \frac{1}{A}\int_0^A m_a\,x\,(d\,z)$$

$$= -\left\{M_2\left(\frac{B^2\,z - \frac{2\,B\,z^2}{2} + \frac{z^3}{3}}{B^2}\right) + M_3\left(\frac{\frac{B\,z^2}{2} - \frac{z^3}{3}}{B}\right) - \frac{1}{B}\int_0^B m_b\,(B - z)\,(d\,z)\right\}$$

In the limit, when x becomes A and z becomes B

$$\frac{M_1\,A}{6} + \frac{M_2\,A}{3} - \frac{1}{A}\int_0^A m_a\,x\,(d\,x) = -\frac{M_2\,B}{3} - \frac{M_3\,B}{6} + \frac{1}{B}\int_0^B m_b\,(B - z)\,(d\,z)$$

On collecting, we have

$$M_1\,A + 2\,M_2\,(A + B) + M_3\,B = 6\left(\frac{\int_0^A m_a\,x\,(d\,x)}{A} + \frac{\int_0^B M_b\,(B - z)\,(d\,z)}{B}\right)$$

etc., etc., as in text, page 4.

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